Oblivious Router Policies and Nash Equilibrium

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***Abstract*— Most of congestion control schemes require users to behave in a cooperative way, so that they respect some "social responsible" rules. However, without forcing end users to adopt a centralized mandated algorithm controlling their behavior (which is not advisable), it is not possible to guarantee that they will not act in a selfish manner. Consequently, a fundamental issue is to evaluate the impact of having users that act in such a manner.**

In such a scenario, having a Nash equilibrium guarantees that no selfish user has incentive to unilaterally deviate from its current state (i.e., it guarantees that we are in a stable state in the presence of selfish users).

However, here we formally prove that an efficient Nash equilibrium can not be reached in practice for any oblivious control policy.

I. Introduction

An important issue that has been largely studied is dealing with congestion control schemes. Since many communication systems in our days are based on the principle of sharing a common resource (e.g., a communication link) among different users, one of the main objectives of such schemes is to establish a number of rules guaranteeing that the common resources are shared fairly among users.

However, most of those schemes require users to behave in a cooperative way, so that they respect some "social responsible" rules. Nevertheless, many authors have already noticed that, without forcing end users to adopt a centralized mandated algorithm controlling their behavior (which is not advisable), it is not possible to guarantee that they will not act in a selfish manner. For instance, the TCP control scheme is voluntarily in nature and critically depends on end–user cooperation [1].

Consequently, several authors have already evaluated the impact of having users that act in a selfish manner [2], [3], [4]. An interesting technique to model selfish users consists of using concepts from *game theory*. From a game-theoretic perspective, users are considered as the *game players* and congestion control schemes establish the *game rules*. An important concept in game theory is the Nash equilibrium: in our context, a Nash equilibrium is a scenario where no selfish user has incentive to unilaterally deviate from its current state. Clearly, being in a Nash equilibrium means that we are in a stable state in the presence of selfish users.

a) Related Work: In [5], the author uses a M/M/1 model and shows that, with Markovian arrival rates, the *fair share* allocation scheme is the only that can guarantee Nash equilibrium within a subset of allocation functions called *MAC*. Akella et al. [2] consider TCP and prove that RED does not have a Nash equilibrium. They also use a variation of Choke [6] and, by using simulation, show that a good Nash equilibrium is reached. Garg et al. [3], by using also TCP, show that, in the presence of selfish users, current schemes will inevitably lead to a congestion collapse. They propose a class of service disciplines called DWS that punish misbehaving users and reward congestion avoiding well behaved users.

Dutta et al. [4] consider a scenario where the arrival rate of users is modeled by a Poisson process. They show that, by using oblivious policies, it is not possible to reach an efficient and fair Nash equilibrium. Observe that, in this context, fair means that all users obtain the same reward. Our paper is deeply inspired in this work and can be considered to be an extension of it, where our contribution is the formalization of some of the claims presented there.

In the next section we introduce our theoretic model. In Section III we present a feature that has to be fulfilled by the aggregate load at equilibrium satisfying an efficient Nash condition. This result is used in Section IV to show that the efficient equilibrium of any oblivious efficient policy can not be reached in practice. Finally, in Section V we present our conclusions and discuss

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about future work.

II. A Game–Theoretic Model

Game theory is a tool for analyzing the interaction of decision makers with conflicting interests. Roughly speaking, a *game* has three components: a set of players, a set of possible actions for each player, and a set of utility functions.

In our system, players are end–point traffic agents. Those agents are selfish, i.e., they are only concerned about their own good. Each player has a strategy which is to control the traffic that the player injects to the network. Currently, TCP traffic is the dominant traffic in the internet and its selfishness can be controlled by the two AIMD parameters [2]. However, this only covers a subset of selfish players (e.g., it does not covers UDP traffic). For this reason, following a approach similar to [4], we model the traffic arrival rate of player i by a Poisson process with average rate λ_i (for all players in the system).

The rules of the games are determined by the queue management policy in routers. Here, we only consider *oblivious* policies, i.e., policies that do not differentiate between packets belonging to different flows. Such type of policies do not consider the current status of the network, but rather, its average status. For this reason, they are very important because of its ease of implementation and deployment. For instance, FIFO, the most commonly used policy, is oblivious.

In general, a user's utility depends on its goodput, loss rate and end–to–end delay. However, for a majority of applications the goodput is the most important factor determining the user's utility [3]. Therefore, we also assume that the utility function of each player is equal to its goodput μ_i. Taking into account that we are considering oblivious policies, we have that $\mu_i = \lambda_i(1 - p(\lambda))$ [7], where $p(\lambda)$ is the drop probability due to an average aggregate load of $\lambda \equiv \sum_{i=1}^{N} \lambda_i$ and an average service time of unity (N denotes the number of players). In order to simplify the analysis, and without loss of generality, in the rest of the paper we assume that the service rate of the system is normalized to 1.

b) Nash Equilibria: In a Nash equilibrium, no player can increase his goodput by either increasing or decreasing their input rate (throughput). Thus, the following condition must be satisfied

$$\left.\frac{\partial \mu_i}{\partial \lambda_i}\right|_{\lambda^*} = 0, \qquad i = 1, \ldots, N, \tag{1}$$

where λ^* is the average aggregate load at equilibrium. This condition can be rewritten as

$$q(\lambda^*) + \lambda_i^* q'(\lambda^*) = 0 \text{ where } q(\lambda) \equiv 1 - p(\lambda). \tag{2}$$

Nonetheless, we are interested in a *symmetric equilibrium*, which imposes $\lambda_i^* = \lambda^*/N$. Hence the Nash condition becomes

$$q(\lambda^*) + \frac{\lambda^*}{N} q'(\lambda^*) = 0. \tag{3}$$

Remark that this symmetry condition implies that the goodput at equilibrium is the same for all players, which is the only way to guarantee that the obtained policy if fair.

c) Efficiency: On the other hand, given a solution for the Nash condition, it is also desirable that such solution has a good efficiency. It is said that a solution is *efficient* when the aggregate goodput at equilibrium μ^*, which is defined as

$$\mu^* \equiv \sum_{i=1}^{N} \mu_i^* = \sum_{i=1}^{N} \lambda_i^* q(\lambda^*) = \lambda^* q(\lambda^*), \tag{4}$$

verifies that $\lim_{N\to\infty} \mu^*$ is a positive constant.

d) Sensitivity: Observing Eq. 3 we may remark that λ^* is, in general, a function of the number N. Hence, the load offered by any of the players at equilibrium λ_i^* also depends on N. In this situation, it is interesting to define a parameter measuring the increase on λ_i^* when N changes. With this purpose and similar to [4], we introduce the *sensitivity coefficient* $\Delta_i(N)$ which can be defined as

$$\Delta_i(N) = \lambda_i^*(N) - \lambda_i^*(N-1). \tag{5}$$

Observe that $\Delta_i(N)$ is a measurement of how difficult is for player i to reach a new equilibrium when the number of users increases from $N-1$ to N. For practical purposes, it will be interesting to obtain oblivious policies having no sensitivity to N ($\Delta_i(N) = 0$). We say that a policy is *reachable* in a practical situation if it has no sensitivity to N. This would guarantee that, once all hosts have reached the equilibrium, they will be able to maintain it without the need of passing a transient period of time searching their new Nash conditions. Hence, in a practical point of view, given that N changes rapidly in a real Internet situation, having $\Delta_i(N) \neq 0$ means that the system would be all the time out of the equilibrium.

e) Our Work: In [4] it has been shown that whereas some policies do not impose a Nash equilibrium (e.g., Drop-tail queueing or RED), there are some others that guarantee it (e.g., VLRED). Furthermore, from the latter type of policies, some cannot impose the existence of an efficient Nash equilibrium (e.g. VLRED) while others guarantee an efficient equilibrium (e.g., EN-AQM). Analogously and taking into account the set of policies that impose a Nash equilibrium, it has been also shown that some of them are very sensitive (e.g., EN-AQM). We note that it is possible to define policies having $\Delta_i(N) = 0$ for all i and N. For example, it can be shown that having an oblivious drop probability of $p(\lambda) = 1 - e^{-\lambda}$, the sensitivity coefficient $\Delta_i(N)$ becomes zero for all i and N. Proving this is immediate just by applying the Nash condition and verifying that λ_i^* is a constant value independent of N.

From the above paragraph, we have that there are policies that impose a Nash equilibrium which are efficient and reachable. However, our goal is to find a policy being efficient and reachable at the same time. For instance, the EN-AQM policy (which is efficient) has been shown in [4] not to be reachable. Similarly, that policy having an oblivious drop probability of $p(\lambda) = 1 - e^{-\lambda}$ (which is reachable) is not efficient since the aggregate goodput at equilibrium $\mu^* = N(1 - e^N)$ goes to zero when N increases. In the following sections, it will be proved that any efficient oblivious policy is sensitive to the number of agents, which makes it unreachable for practical purposes.

At this point, we would like to remark that Dutta et al. have provided a result somehow similar in [4]. However, they assume that $p(\lambda^*)$ must be a non-decreasing and convex function. Furthermore, they assume that their sensitivity coefficient (defined as $\lambda^*(N) - \lambda^*(N-1)$ is N^α. Such assumptions, although simplify the proof, are arbitrary. On the contrary, our result is completely general. Surprisingly, we also prove that their assumption about the sensitivity coefficient (i.e., that must be of the form N^α) constitutes a sufficient condition to obtain an efficient solution.

III. The Nash Condition in the Continuum Limit

As it has been stated previously, the average aggregate load at equilibrium λ^* derived from the Nash condition depends on N, the number of agents involved in the network. Hence, λ^* is a discrete function $\lambda^* : \mathbb{N} \rightarrow \mathbb{R}^+$, which for every value of N returns the λ^* imposed by the Nash condition for N agents.

However, although λ^* is a discrete function of N, it is always possible to regard λ^* as a twice derivable function $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that $f(N) = \lambda^*(N)$ for all integer N. Therefore, Eq. 3 can be seen as the following condition, which holds for all $v \geq 1$,

$$q[f(v)] + \frac{f(v)}{v} q'[f(v)] = 0, \qquad (6)$$

where, according to the definition of f, the derivative must be understood as $()' \equiv \frac{d}{df}()$. For simplicity, let us denote $q(v)$ instead of $q[f(v)]$.

Consequently, if it is used the notation $\dot{()} \equiv \frac{d}{dv}()$, the Nash condition in the continuum limit is written as

$$q(v) + \frac{f(v)}{v} \frac{\dot{q}(v)}{\dot{f}(v)} = 0, \qquad (7)$$

Then, a first order ordinary differential equation is obtained,

$$-\frac{\dot{f}(v)}{f(v)} v \; dv = \frac{dq(v)}{q(v)}, \qquad (8)$$

whose solution can be written formally as follows,

$$q(v) = \mathrm{D} \; \mathsf{e}^{-I(v)}, \qquad (9)$$

where D is a constant of integration and $I(v)$ is defined in this manner

$$I(v) \equiv \int \frac{\dot{f}(v)}{f(v)} v \; dv. \qquad (10)$$

Clearly, $q(v)$ must be a well defined probability (i.e., it must be in the range $[0, 1]$). The next lemma (whose proof we omit for space reasons) shows the form of $f(v)$ to fulfill this.

Lemma 1: $q(v)$ is a well defined probability if and only if the function $f(v)$ can be written as

$$f(v) = \mathrm{D'} \; \mathsf{e}^{J(v)},, \qquad (11)$$

with D'> 0 and

$$J(v) \equiv \frac{I_+(v)}{v} + \int_1^v \frac{I_+(z)}{z^2} \; dz, \qquad (12)$$

being $I_+(v)$ such that $I(v) \equiv -M + I_+(v)$, with $0 \leq M < \infty$ and $I_+(v) \geq 0$ for all $v \geq 1$.

Now, we use that result to demonstrate that any efficient solution $f(v)$ satisfying the Nash condition must verify that $\lim_{v\to\infty} f(v) = f_\infty$, with $f_\infty \in (0, \infty)$.

Lemma 2: Any efficient $f(v)$ that verifies the Nash condition must verify that $0 < \lim_{v\to\infty} f(v) < \infty$.

Proof: The lemma is proved in two steps. First, we present the restriction imposed on $f(v)$ by the Nash condition. Second, we present the restriction imposed on $f(v)$ by having an efficient solution to the Nash condition.

- *The restriction imposed by the Nash condition:* Since both terms in the right side of Equation 12 are non–negative, then we have that $J(v) \geq 0$ for all $v \geq 1$. Assuming that $f(v)$ verifies the Nash condition, from Equation 11 we have that $f(v) \geq$ D' for all $v \geq 1$. Since $D' > 0$ then $\lim_{v\to\infty} f(v) > 0$.
- *The restriction imposed by having an efficient solution to the Nash condition:* By *reductio ad absurdum*. Assume that there exists an efficient solution

to the Nash condition and prove that such solution cannot be satisfied by an average aggregate load $f(v)$ verifying $\lim_{v\to\infty} f(v) = \infty$.

If $\lim_{v\to\infty} f(v) = \infty$, then, in order to obtain a function $q(v)$ satisfying the Nash condition, we have that $\lim_{v\to\infty} J(v) = \infty$. If such a solution is also efficient, the following condition must hold

$$0 < \lim_{v\to\infty} \lambda^*(v)\ q[\lambda(v)] < \infty,$$

which in the continuum limit implies that

$$0 < \lim_{v\to\infty} f(v)\ \mathrm{e}^{-I_+(v)} < \infty$$

Taking into account Eq. 11, the former condition can be written as

$$0 < \lim_{v\to\infty} \mathrm{e}^{J(v)}\ \mathrm{e}^{-I_+(v)} < \infty$$

which can only be true if

$$\lim_{v\to\infty} J(v) - I_+(v) \neq \pm\infty. \tag{13}$$

Since $\lim_{v\to\infty} J(v) = \infty$, Eq. 13 can only be satisfied if $\lim_{v\to\infty} I_+(v) = \infty$. But, if it is verified that $\lim_{v\to\infty} J(v) - I_+(v) =$ constant, with $\lim_{v\to\infty} J(v) = \infty$ and $\lim_{v\to\infty} I_+(v) = \infty$, it is easy to check that

$$\lim_{v\to\infty} \frac{J(v)}{I_+(v)} = 1,$$

which implies (by definition of $J(v)$) that

$$\lim_{v\to\infty} \frac{1}{I_+(v)} \int_1^v \frac{I_+(z)}{z^2}\ dz = 1. \tag{14}$$

Define the function $s(v)$ as follows:

$$s(v) \equiv \frac{1}{I_+(v)} \int_1^v \frac{I_+(z)}{z^2}\ dz. \tag{15}$$

We have that Eq. 14 imposes on $s(v)$ the condition $\lim_{v\to\infty} s(v) = 1$. Since it is also true that $\lim_{v\to\infty} 1/s(v) = 1$, there exists some $V \geq 1$ such that, for all $v > V$,

$$\frac{1}{s(v)} < 1 + \epsilon. \tag{16}$$

where ϵ is a positive real number.

On the other hand, by definition of $s(v)$, its derivative with respect to v is the following

$$\frac{I_+}{v^2} = \dot{s}(v) I_+ + s(v)\dot{I}_+. \tag{17}$$

Then, the following ordinary differential equation is obtained

$$\frac{\dot{I}_+}{I_+} = \frac{1}{s(v)v^2} - \frac{\dot{s}(v)}{s(v)}, \tag{18}$$

whose solution can be written formally as

$$I_+(v) = \frac{\mathrm{C}}{s(v)}\ \exp\left[\int \frac{dv}{s(v)v^2}\right], \tag{19}$$

where C is a constant of integration.

If Eq. 16 is taken into account, we have that, for all $v > V$,

$$I_+(v) < \mathrm{C}\ (1+\epsilon)\ \exp\left[\int \frac{1+\epsilon}{v^2}\ dv\right] = \mathrm{C}\ (1+\epsilon)\ \exp\left[\mathrm{C'} - \frac{1+\epsilon}{v}\right], \tag{20}$$

where C' is a constant of integration. As a consequence, it is derived that, for all $v > V$

$$I_+(v) < \mathrm{C}\ (1+\epsilon)\ \exp\left[\mathrm{C'} - \frac{1+\epsilon}{V}\right] = \mathrm{C''}, \tag{21}$$

where C” is a constant. Therefore, it is obtained that $\lim_{v\to\infty} I_+(v) < \infty$. However, this contradicts the result derived from Eq. 13 (i.e., $\lim_{v\to\infty} I_+(v) = \infty$). Then, we conclude that any average aggregate load $f(v)$, such that $\lim_{v\to\infty} f(v) = \infty$, cannot result in an efficient solution to the Nash condition. ■

IV. Efficient Solutions to the Nash Condition

As it has been shown in Lemma 2, the efficient solutions to the Nash condition must be searched among the average aggregate loads $f(v)$ such that $\lim_{v\to\infty} f(v) = f_\infty$, with $f_\infty \in (0, \infty)$. This case can always be written as $f(v) \equiv f_\infty[1 + \tilde{f}(v)]$, where $\tilde{f}(v) > -1$ for all $v \geq 1$ and $\lim_{v\to\infty} \tilde{f}(v) = 0$. Then, $I(v)$ can be written in terms of $\tilde{f}(v)$ as follows[1]

$$I(v) = \text{cons} + \int_1^v \frac{\dot{\tilde{f}}(z)}{1+\tilde{f}(z)} z\ dz. \tag{22}$$

On the first hand and from the mean-value theorem for integrals [8], it is derived that $I(v)$ is a constant for all $1 \leq v < \infty$ (it is the integration of a continuous function in a finite interval). Hence, in order to obtain a solution to the Nash condition, it is not necessary to verify that $I(v) > -\infty$ for all $v \geq 1$ but to check that $\lim_{v\to\infty} I(v) \neq -\infty$.

On the other hand, since $\lim_{v\to\infty} f(v) = f_\infty$, the condition of efficiency is verified when $\lim_{v\to\infty} q(v) \neq 0$. Then, from Eq. 9 is deduced that the efficiency is guaranteed if and only if $\lim_{v\to\infty} I(v) \neq \infty$.

[1]Notice that $\frac{\dot{\tilde{f}}(z)}{1+\tilde{f}(z)} z$ is a continuous function, since the average aggregate load $f(v)$ was defined as twice derivable (recall that there exists this freedom when the continuum limit is taken).

Therefore, there exists an efficient solution to the Nash condition if and only if the condition $\lim_{v\to\infty} I(v) \neq \pm\infty$ holds. That is, if and only if

$$\int_1^{\infty} \frac{\dot{\tilde{f}}(v)}{1+\tilde{f}(v)} v \; dv \neq \pm\infty. \qquad (23)$$

In this section it is shown that, in general, not all average aggregate load $f(v)$ such that $\lim_{v\to\infty} f(v) = f_\infty$ satisfies Eq. 23. However, it will be demonstrated that there are two conditions which define the set of efficient solutions.

Theorem 1 (Sufficient Condition): If $\tilde{f}(v)$ is a twice derivable function which behaves asymptotically as

$$\dot{\tilde{f}}(v)v \sim \frac{1}{v^{1+\alpha}} \text{ with } \alpha > 0. \qquad (24)$$

then, an efficient solution is derived.

Proof: By definition of $\tilde{f}(v)$, it is easy to check that

$$\lim_{v\to\infty} \frac{1}{1+\tilde{f}(v)} = 1. \qquad (25)$$

Thus, given any $\epsilon > 0$, there exists some $V \geq 1$ such that, for all $v > V$,

$$\left| \frac{1}{1+\tilde{f}(v)} \right| < 1 + \epsilon. \qquad (26)$$

Then, it is deduced that

$$\left| \int_1^{\infty} \frac{\dot{\tilde{f}}(v)}{1+\tilde{f}(v)} v \; dv \right| < \int_1^{\infty} \left| \frac{\dot{\tilde{f}}(v)}{1+\tilde{f}(v)} v \right| dv$$
$$< \text{B} + (1+\epsilon) \int_V^{\infty} |\dot{\tilde{f}}(v)| v \; dv,$$

where B is a real number defined as

$$\int_1^{V} \left| \frac{\dot{\tilde{f}}(v)}{1+\tilde{f}(v)} v \right| dv. \qquad (27)$$

If $\dot{\tilde{f}}(v)v \sim v^{-(1+\alpha)}$ with $\alpha > 0$, and taking into account that the integration of a function which asymptotically behaves as $v^{-(1+\alpha)}$ is a function which tends to zero as $v \to \infty$, it is straightforward to check that

$$\int_V^{\infty} |\dot{\tilde{f}}(v)| v \; dv = \text{B'} \text{ with } \text{B}' \text{ being a constant}. \qquad (28)$$

Thus, it is deduced the condition given by Eq. 23. Namely,

$$\left| \int_1^{\infty} \frac{\dot{\tilde{f}}(v)}{1+\tilde{f}(v)} v \; dv \right| < \text{B} + (1+\epsilon)\text{B'} < \infty. \qquad (29)$$

which implies that there exists an efficient solution. ∎

Theorem 2 (Necessary Condition): If the Nash condition is verified efficiently, the following equation holds

$$\lim_{v\to\infty} \dot{\tilde{f}}(v)v^2 = 0. \qquad (30)$$

Proof: Let us suppose that $\lim_{v\to\infty} \dot{\tilde{f}}(v)v^2 =$ A' > 0. Then, for every A"$\in$ (0,A'), it is always possible to find a $V_1 \geq 1$ such that $\dot{\tilde{f}}(v)v^2 \geq$A", for all $v > V_1$. On the other hand, $\lim_{t\to\infty} \tilde{f}(v) = 0$. Thus for every $\epsilon \in (0,1)$ there exists a $V_2 \geq 1$ such that

$$\frac{1}{1+\tilde{f}} > 1 - \epsilon, \qquad (31)$$

for all $v > V_2$. Hence, it is deduced from Eq. 23 that, for all $v > \max\{V_1, V_2\}$,

$$\text{cons} = \int_1^{\infty} \frac{\dot{\tilde{f}}(v)v^2}{1+\tilde{f}(v)} \frac{dv}{v} \geq \qquad (32)$$
$$\text{A} + \text{A"} \int_V^{\infty} \frac{1}{1+\tilde{f}(v)} \frac{dv}{v} \geq$$
$$\text{A} + \text{A"}(1-\epsilon) \int_V^{\infty} \frac{dv}{v} = \infty,$$

where A is defined as

$$\text{A} \equiv \int_1^{V} \frac{\dot{\tilde{f}}(v)}{1+\tilde{f}(v)} v \; dv. \qquad (33)$$

Consequently, it is a constant. This is a contradiction which arises because it was assumed that $\lim_{v\to\infty} \dot{\tilde{f}}(v)v^2 > 0$.

Similarly, when $\lim_{t\to\infty} \dot{\tilde{f}}(v)v^2 = -$A' < 0, for every A"$\in$ (0,A'), there exists some $V_1 \geq 1$ such that $\dot{\tilde{f}}(v)v^2 \leq -$A", for all $v > V_1$. In addition, $\lim_{v\to\infty} \tilde{f}(v) = 0$. Thus given any $\epsilon > 0$ there exists some $V_2 \geq 1$ such that

$$\frac{1}{1+\tilde{f}} < 1 + \epsilon, \qquad (34)$$

for all $v > V_2$. Hence, it is derived from Eq. 23 that, for all $v > \max\{V_1, V_2\}$,

$$\text{cons} = \int_1^{\infty} \frac{\dot{\tilde{f}}(v)v^2}{1+\tilde{f}(v)} \frac{dv}{v} \leq \qquad (35)$$
$$\text{A} - \text{A"} \int_V^{\infty} \frac{1}{1+\tilde{f}(v)} \frac{dv}{v} \leq$$
$$\text{A} - \text{A"}(1+\epsilon) \int_V^{\infty} \frac{dv}{v} = -\infty.$$

Such contradiction only disappears when the condition $\lim_{v\to\infty} \dot{\tilde{f}}(v)v^2 < 0$ is rejected.

Notice that it is only possible to find an A"> 0 when A' is not zero. If A' is zero the former reasoning fails because the first case results in cons$< \infty$ and the second one derives in cons$> -\infty$ and nothing can be argued.

The previous results imply that, given an efficient solution to the Nash condition, it is not feasible that $\lim_{v\to\infty} \ddot{\tilde{f}}(v)v^2 \neq 0$. But, it could be possible that the condition $\lim_{v\to\infty} \ddot{\tilde{f}}(v)v^2 = 0$ was also rejected. In that case, it would derive that there is no twice derivable $\tilde{f}$ verifying Eq. 23. That is, there is no efficient solution to the Nash condition.

However, in the previous theorem was demonstrated that the functions $\tilde{f}(v)$ which behave asymptotically as $\dot{\tilde{f}}(v)t \sim v^{-(1+\alpha)}$, with $\alpha > 0$, are efficient solutions. But these functions verify that $\lim_{v\to\infty} \ddot{\tilde{f}}(v)v^2 = 0$, thus this condition defines a non-empty set of efficient solutions. Therefore, it can be affirmed that $\lim_{v\to\infty} \ddot{\tilde{f}}(v)v^2 = 0$ is a necessary condition to obtain an efficient solution to the Nash condition. ■

Discussion

As it can be readily seen, the sufficiency condition (Theorem 1) means that the set of efficient solutions is bigger than the set of functions which asymptotically behave as $\dot{\tilde{f}}(v)v \sim v^{-(1+\alpha)}$ with $\alpha > 0$. On its hand, the necessity condition (Theorem 2) means that the set of efficient solutions is smaller than the set of functions which verify that $\lim_{v\to\infty} \ddot{\tilde{f}}(v)v^2 = 0$.

From a practical point of view this means that the set of efficient solutions are functions such that asymptotically behave as $f(v) \sim v^{-(1+\alpha)}$ with $\alpha > 0$. Consequently, this implies that any oblivious efficient policy must have an aggregate offered load at equilibrium (λ^*) falling asymptotically to a positive constant (c) when the number of users (v) increases.

In terms of the offered load, this result tells us that, at equilibrium, any efficient solution must have a λ^* which falls with v to a constant value at least as $\frac{1}{v^{1+\alpha}}$. As our equilibrium is assumed to be symmetric, this implies that, in the asymptotic limit, the load offered by any of the hosts at equilibrium changes with v in the form $\lambda_i^*(v) \sim \frac{c}{v}$. Hence, the sensitivity coefficient for any host behaves like $\Delta_i(v) \sim \frac{c}{v} - \frac{c}{v-1} \sim -\frac{c}{v^2}$ in that limit.

This allows us to conclude that, in situations where the number of sessions changes rapidly (which are very realistic situations), the efficient equilibrium of any oblivious efficient policy is not easily reachable, because the offered load of hosts depend strongly on the number of current sessions.

V. Conclusions

In this paper it has been shown that when using oblivious efficient policies of buffer management, hosts cannot arrive properly at a stable equilibrium point (at least, without any other additional mechanism). Such a result is completely general since we have not made any assumption, either about the behavior of $p(\lambda^*)$ or about the behavior of the sensitivity coefficient.

Currently, we are involved in making a similar analysis but, instead of taking as user's utility its goodput, taking the end–to–end delay to define it. Also, we are considering introducing the loss rate, in combination with either the goodput and the end–to–end delay, to define the user's utility.

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