Isochronous synchronization in mutually coupled chaotic circuits

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This paper examines the robustness of isochronous synchronization in simple arrays of bidirectionally coupled systems. First, the achronal synchronization of two mutually chaotic circuits, which are coupled with delay, is analyzed. Next, a third chaotic circuit acting as a relay between the previous two circuits is introduced. We observe that, despite the delay in the coupling path, the outer dynamical systems show isochronous synchronization of their outputs, i.e., display the same dynamics at exactly the same moment. Finally, we give here the first experimental evidence that the central relaying system is not required to be of the same kind of its outer counterparts. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2737820]

During the past years there has been a great interest in understanding the dynamics of chaotic systems and, specifically, how synchronization between them arises. The way of introducing the coupling between chaotic systems is crucial to obtain different kinds of synchronization. In the current paper we show that not only the type of coupling but the number of dynamical units is relevant in order to observe two different types of synchronization, namely, achronal and isochronous synchronization. When two identical chaotic circuits are bidirectionally coupled, it is possible to obtain synchronous behavior if internal (and coupling) parameters are adequately tuned. Nevertheless, there always exists a time delay between both synchronized outputs, which corresponds to the time taken by the signal to travel from one circuit to the other. In the current work we show how it is possible to obtain isochronous synchronization, i.e., both circuits with the same output at exactly the same moment, by introducing a third dynamical unit between both chaotic circuits. Surprisingly, the relaying unit does not need to be identical to those to be synchronized and even it can be a completely different dynamical system.

I. INTRODUCTION

Chaos synchronization was initially focused in unidirectionally coupled systems.[1] The reason beyond this fact could be that, for technical applications, it is interesting to reproduce the state of a certain chaotic system, no matter the distance or the number of the replica systems. This kind of configuration is commonly known as master-slave configuration[2] and is the most extended technique to synchronize chaotic systems.[3] Most of the applications were achieved in the chaotic communications field, where a chaotic transmitter hides a secret message that is recovered at the receiver when the latter synchronizes with the chaotic part of the received input,[4,5] i.e., reproducing the state of the transmitter. Nevertheless, in nature, bidirectional coupling is also present, as it can be seen in predator/prey networks,[6] interactions between individuals in a social network,[7] or swarm dynamics.[8] Recently, a combination of unidirectional and bidirectional coupling has been proposed as a technique of bidirectional communication with chaotic carriers.[9]

Without regard to the direction of the coupling, the interaction between two chaotic systems has been deeply studied during the past decade, focusing on the ability of synchronization even in the presence of noise or delay.[3] More recently, the study of complex networks has dealt with the synchronization of large communities of chaotic systems, where delay between interacting units is considered.[10,11] Less attention has been paid to the transition from the simplest case, i.e., two bidirectionally coupled systems, to a broad community of chaotic oscillators.

Here we depart from two bidirectionally chaotic systems and show a counterintuitive phenomenon that arises when a third chaotic element placed between them is considered: *the isochronous synchronization of the two outer chaotic systems*. This fact has been recently reported in bidirectionally coupled semiconductor lasers,[12,13] where a third laser, in this case, is also requested. The present work follows the path opened by Fischer *et al.*[12] and goes one step beyond. First, we analyze the robustness of the phenomenon for different delay times, showing that accurate values of the delay time are not required. Second, we give *the first experimental evidence that the relaying system could be different from those to be synchronized at zero-lag*. The manuscript is organized as follows: In Sec. II, we study the synchronization of two mutually coupled chaotic circuits, with a certain delay in the coupling path. We show that zero-lag synchronization is not observed in this particular configuration. In Sec. III, we introduce a relay system between the two chaotic circuits and observe the appearance of isochronous (zero-lag) synchronization between the outer units. Finally, in Sec. IV we show, with an example, that zero-lag synchronization holds even when a different dynamical system is used as the relay system, ending with some concluding remarks.

II. MUTUALLY COUPLED CHAOTIC CIRCUITS

Unidirectional synchronization of chaotic circuits and specifically, Chua circuits, have been deeply studied during

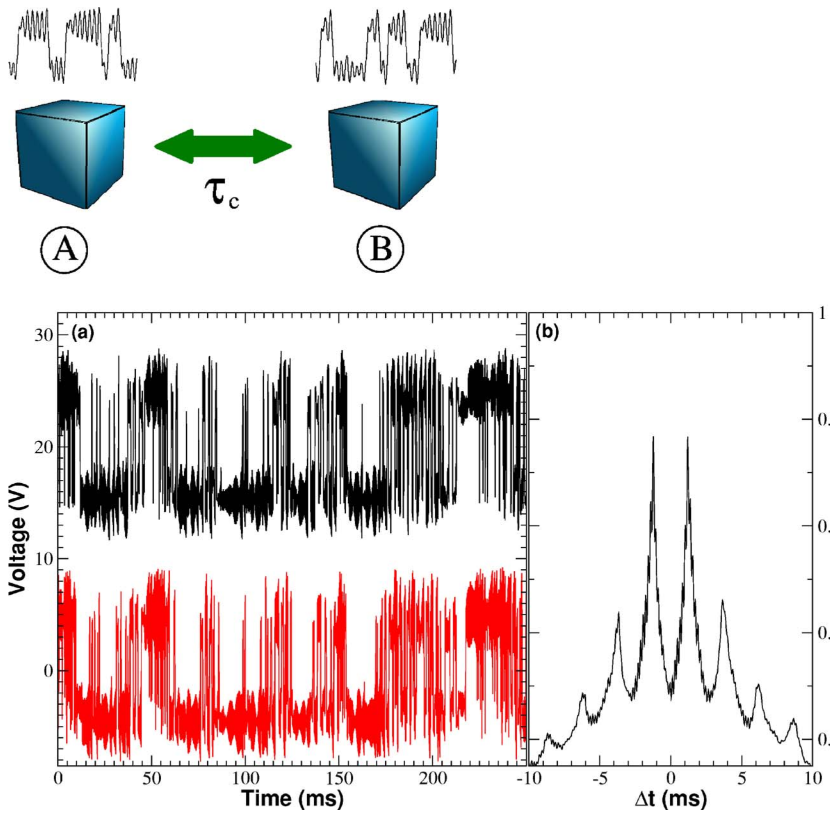

FIG. 1. (Color online) (Upper) Qualitative description of the experimental setup. Two similar chaotic systems (A and B) are coupled through a bidirectional channel with a time delay τ_c. (Lower) In (a) we plot the output voltage V_1^A (above) and V_1^B (below), which has been vertically shifted to ease comparison. In (b) the cross-correlation function is plotted, showing two maxima of similar value at a time delay $\Delta t = \pm \tau_c$. For this particular example, the internal coupling parameters are $R_{\text{coup}} = 47\ \text{k}\Omega$ and $\tau_c = 1.1625$ ms.

the past years. From two circuits to a chain of many of them, synchronization has been reported under different experimental setups. In a general framework, we can distinguish between different types of synchronization if we consider the system that is leading the dynamics along with the delay between the outputs of the synchronized systems. In lag synchronization,[15] for example, the receiver system follows the evolution of the transmitter with a delay τ due to a parameter mismatch. In achronal synchronization, there is a time lapse between the output of the synchronized systems, which is a consequence of a certain delay in the transmission line. More recently, a counterintuitive phenomenon has been reported, the anticipated synchronization, where the receiver system advances in time the signal of the transmitter.[16,17]

In all cases where unidirectional injection is considered, a leader and follower role can be distinguished, being that it is the former in the system that sends the signal to the other. However, this reasoning does not apply for the case of mutually (bidirectional) coupled systems. In this condition, the leader and follower roles can only be inferred from the analysis of the circuit outputs. When two system are considered, both circuits affect each other and eventually synchronize, which leads any of them to assume the role of the leader (or follower).

Here we are interested in the synchronization between two chaotic electronic circuits when bidirectional coupling with delay is considered. Several works have dealt with mutually coupled chaotic circuits;[18–22] nevertheless, less attention has been paid to electronic circuits coupled with delay. For the case of two chaotic systems, lasers have been the paradigmatic example of coupled systems with delay.[24–27] The seminal work of Heil *et al.*[24] has shown the influence of the non-negligible coupling time between two mutually coupled lasers. In particular, synchronization between two chaotic lasers was observed with a time delay τ_c, corresponding to the time for the output signal to travel between the two dynamical systems. Furthermore, an alternation between the leader and the follower was observed, i.e., there was not a clear leader (follower) in the dynamics.

The experimental setup studied here is schematically represented in Fig. 1. The output of two chaotic Chua circuits is connected bidirectionally through a transmission line with a delay τ_c, which means that the output signal needs some time to arrive to the other circuit. Details of the Chua circuit and electronic connections between them are given in Appendices A and C, respectively. The output of both circuits are chaotic when uncoupled and keep their dynamics for low to moderate coupling strengths. Nevertheless, when the coupling strength crosses a certain threshold, synchronization arises. Figure 1(a) shows the output voltage of both circuits for a coupling resistance of $R_c = 47\ \text{k}\Omega$, which corresponds to a moderate coupling. We can observe how both signals show a relatively good synchronization. The quality of the synchronization is measured with the cross-correlation function, which gives an estimate of the similarity between two time series shifted with a time lag Δt. The cross-correlation function between two output voltages V_A and V_B is defined as

$$C(\Delta t) = \frac{\langle (V_A(t) - \langle V_A \rangle)(V_B(t+\Delta t) - \langle V_B \rangle)\rangle}{\sqrt{\langle (V_A(t) - \langle V_A \rangle)^2 \rangle \langle (V_B(t) - \langle V_B \rangle)^2 \rangle}}, \quad (1)$$

where Δt is a temporal shift introduced in V_B and the brackets represent time averaging. This tool helps to find the delay between two time series, which corresponds to the Δt with

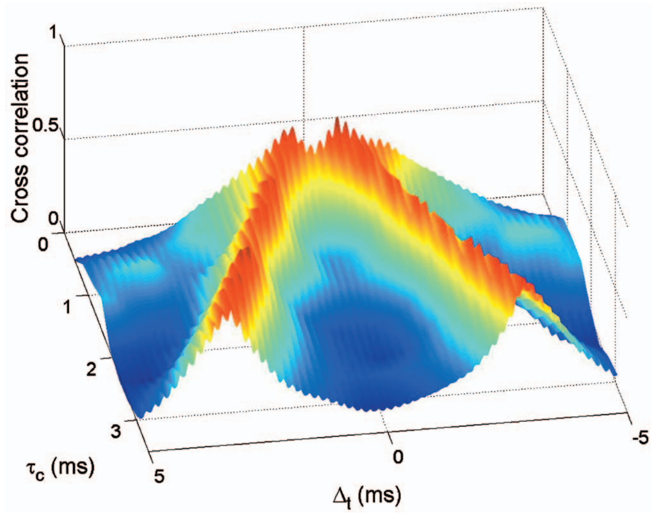

FIG. 2. (Color) Cross-correlation plot of Chua circuits A and B, as a function of the time shift Δt between both series and the time delay τ_c. We can observe how the highest correlation ($C_{max} \sim 0.64$) always occurs at $\Delta t = \pm \tau_c$, which indicates that we have lag synchronization with exchanges in the leader-follower roles between both circuits.

the highest correlation $(-1 < C(\Delta t) < 1)$. In Fig. 1(b) we plot the cross correlation between the output voltage of both circuits, which have two maxima at precisely the coupling time $\pm \tau_c$. It is worth mentioning that both maxima have similar values ($C \sim 0.77$) indicating that there is not a clear leader or follower in the dynamics, i.e., both circuits alternate their role.

At this point we make a systematic study of the influence of the coupling time in the synchronization of both circuits, since phenomena such as amplitude death,[28] symmetry breaking,[24] or periodic regimes[26] have been previously reported in experiments with mutually coupled systems. With a fixed coupling strength, we sweep the coupling time τ_c and check the quality of the synchronization between both circuits. Figure 2 shows the cross correlation as a function of τ_c. We observe how the highest cross correlation is always obtained at $\pm \tau_c$, which indicates, first of all, that the delay between both outputs matches the coupling time and, second, that the switching of the leader-follower role is independent of τ_c. Therefore, we can say that the phenomenon is robust against the coupling time and, furthermore, that isochronous synchronization, i.e., zero lag between circuit outputs, is not observed in two bidirectionally coupled circuits with delay, a fact previously reported in chaotic lasers.[24]

III. ISOCHRONOUS SYNCHRONIZATION

Arriving to this point the question about if it is possible to obtain zero-lag synchronization in mutually coupled chaotic circuits when a delay is considered in the coupling path is still open. A recent work by Fischer *et al.*[12] has shown that the addition of a relay system between two chaotic lasers can lead to isochronous synchronization between the outer systems. With this idea in mind, we introduce a third Chua circuit between the two previous ones, keeping the bidirectional coupling and the delay in the transmission channels. Figure 3 shows the experimental setup, where the intermediate Chua circuit, which acts as the relay system, is drawn in red since it has different internal parameters from those of the outer Chua circuits (see Fig. 3 for details). The coupling time and the coupling strength are set to be equal at all paths, leading to a symmetrical system. In Fig. 3(a) we plot the output voltages V_1 of the three circuits for intermediate coupling ($R_c = 1.2$ kΩ). We can observe how circuits A and C are synchronized at exactly the same time despite the delay in the coupling lines, which is the typical signature of isochronous synchronization.

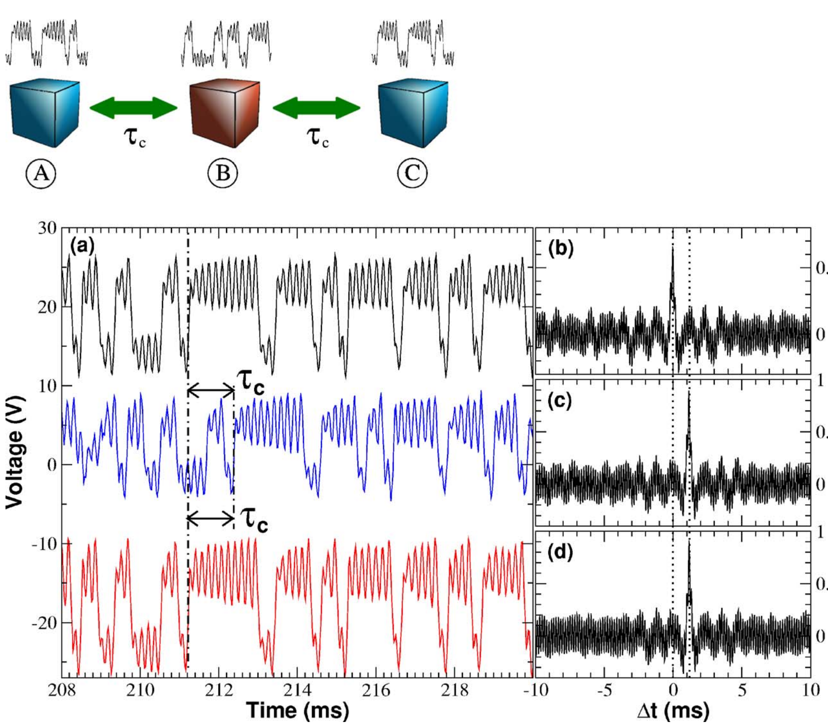

FIG. 3. (Color online) (Upper) Qualitative description of the experimental setup. A third (B) Chua circuit is introduced between A and C. We adjust the internal parameters of the Chua circuit B to be different from those of the outer Chua circuits: For the circuit *A* and *C*, $R_{exc} = 1.85$ kΩ and $L_o = 14$ mH, whereas for circuit *B*, $R_{exc} = 1.76$ kΩ and $L_i = 10$ mH. The three circuits are tuned in the double scroll chaotic regime. (Lower) In (a) we plot the V_1 variable of the circuits (vertically shifted). We can observe how Chua circuits A and C show isochronous synchronization (zero delay) while the central one (B) is lag synchronized with its outer counterparts. (Left) The cross-correlation function between (b) A-C, (c) A-B, and (d) C-B is plotted. We can observe how A and C synchronize with zero delay, while B follows the outer circuits with a delay corresponding to the coupling time τ_c.

An interesting point arises when looking at the output of the relaying system. We can observe how the Chua circuit B is also synchronized with the other two, but in this case it is delayed with a time corresponding to τ_c. In this way, the central system is following the dynamics of the outer circuits and therefore it is not driving them.

Figures 3(b)–3(d) show the cross-correlation function between pairs of circuits. We can observe how for the case of Chua circuits A and B the correlation peak is obtained at zero delay, indicating isochronous synchronization. As expected, correlations of the external circuits (A and C) with the central one (B) show the achronal synchronization, with a peak at exactly the coupling time τ_c. In this case, there is no alternation between the leader role, and the central Chua circuit is always the follower.

In order to show the robustness of the phenomenon versus the coupling time, we repeat the experiment with different delay times from values ranging from near to zero until $\tau_c \sim 3$ ms. The cross-correlation function shows in all cases the zero-lag synchronization for the outer circuits (see Fig. 4, upper plot) and achronal synchronization with regard to the central one (see Fig. 4, bottom plots). It is worth mentioning that the central circuits do not necessarily need to be matched with the outer ones. In fact, as mentioned before, internal parameters of the relay Chua circuit were deliberately detuned.

IV. REPLACING THE RELAY SYSTEM

Since the isochronous synchronization seems to be dependent on the symmetry of the system, it would be reasonable to obtain the same results with a different dynamical system acting as a relay, since symmetry would be preserved (as long as the outer circuits are identical). With this aim, we replace the central Chua circuit by a Sprott circuit,[23] which is a different chaotic electronic circuit. Details of the circuit are given in Appendix B. In Fig. 5 we show a schematic description of the experimental setup, where we can see that, despite the different central unit, the system maintains the symmetry. Figure 5(b) shows the time series of the circuit outputs. We observe how the zero-lag synchronization holds for the Chua circuits. At the same time, the Sprott circuit also synchronizes, in this case, advancing the dynamics of the outer ones a time equal to τ_c. Cross-correlation functions between pairs of circuits quantifies the phenomenon observed in the time series, circuits A and B have a maximum at zero delay, while correlations with the central circuit show that, in this case, the relaying system is leading the dynamics.

We have done several experiments where the central circuit was modified in order to check the influence of the characteristic frequency of the relaying system. Specifically, Chua circuits, Sprott circuits, and Rössler circuits (i.e., a circuit that implements the Rössler equations) were tested, all of them with different internal parameters (in order to modify their frequency). We have observed that isochronous synchronization appears only if the central circuit has a fast enough characteristic frequency (in the absence of coupling), which must guarantee that the transmitted signal is not filtered. Furthermore, we have seen that this kind of synchronization is very sensitive to a mismatch in the coupling in the

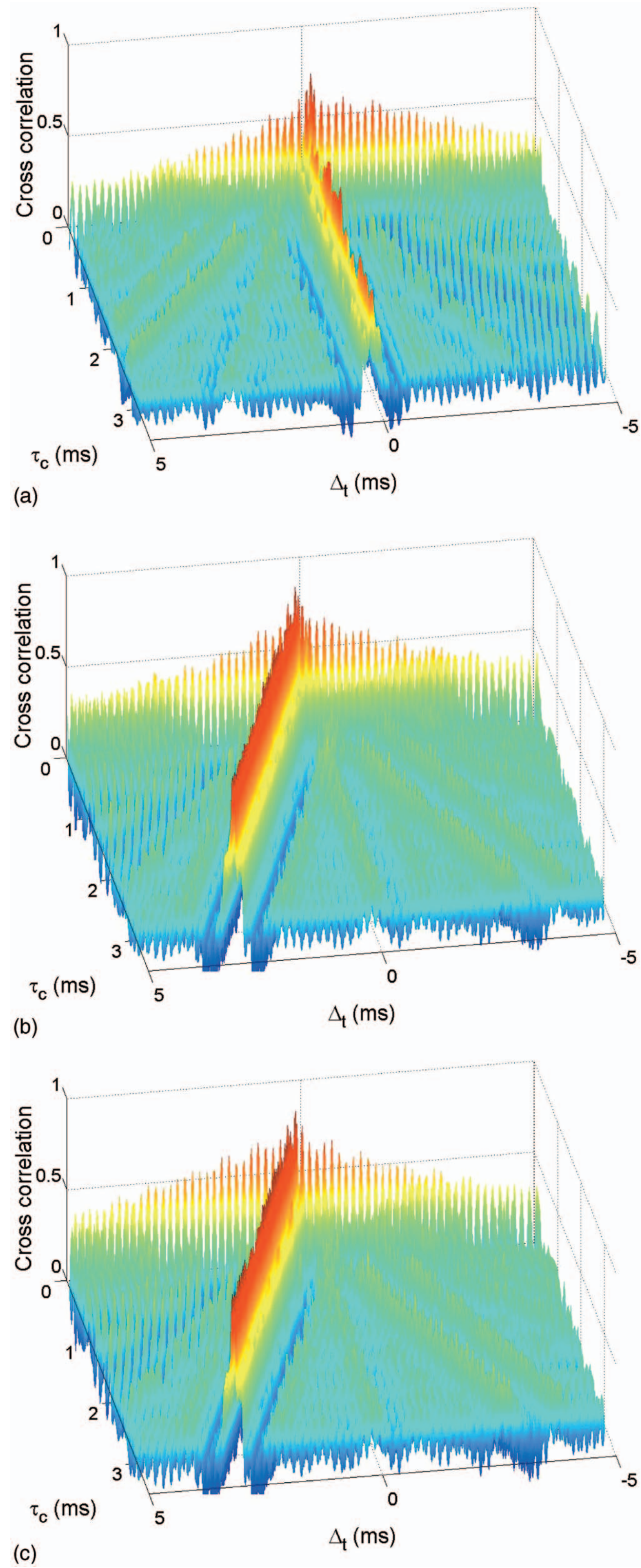

FIG. 4. (Color) Cross-correlation plots of Chua circuits A and C (upper), A and B (bottom left), and C and B (bottom right), as a function of the time shift Δt between both series and the delay in the transmission line τ_c. We can observe isochronous synchronization between A and C, since the best correlation is always reported at zero lag (upper figure). The bottom plots show lag synchronization between the central and outer circuits, since the best correlation is always observed with a delay of τ_c, no matter what its value is.

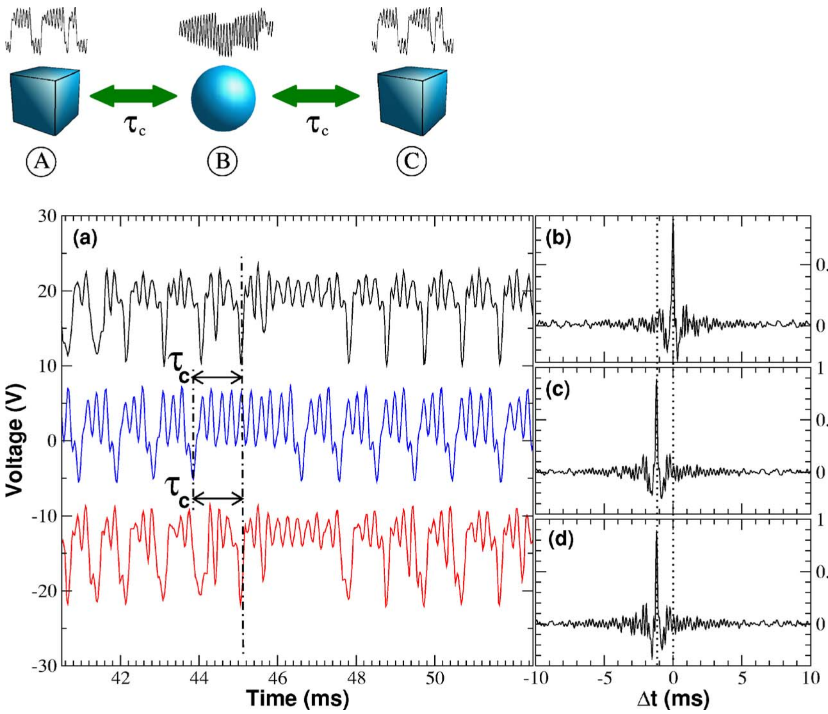

FIG. 5. (Color online) (Upper) Qualitative description of the experimental setup. Two similar chaotic systems A and C (Chua circuits) are coupled through a different chaotic system B (Sprott circuit). In (a) we plot the output voltage of the three circuits showing that despite being different dynamical systems all of them synchronize their dynamics. Furthermore, the two outer circuits keep the zero-lag synchronization, as can be observed both in the time series (a) and in their cross-correlation plots (b). The central circuit B synchronizes with the outers with a delay equal to the coupling time τ_c, despite being a completely different dynamical system. (c) and (d) show the cross correlation between the central and outer circuits, where the central circuit is advanced a time interval equal to the coupling time τ_c.

sense that, when we introduce an asymmetry in the coupling time or coupling strength, zero-lag synchronization is lost. Interestingly, similar phenomena have been reported in interconnected cortical areas of the brain, where simulations based on realistic properties of the neural architecture have shown the appearance of time lags when the symmetry of certain parameter values is lost.[29]

V. CONCLUSIONS

This work is focused on the phenomenon of synchronization in mutually coupled circuits with delay in the coupling connections. First, we analyze the synchronization of two chaotic circuits as a function of the delay time. We observe that despite both circuits are synchronized when they are similar, a time delay between both outputs appears. The delay is equal to the coupling time between both circuits τ_c, i.e., the time needed by the signal to travel from one circuit to the other. Furthermore, the roles of leader and follower in the dynamics are exchanged continuously between both circuits, a phenomenon previously reported in coupled semiconductor lasers.[24] Next, we include a relay circuit between the two chaotic circuits, keeping the bidirectional coupling between all units. Under this configuration, switching in the leader/follower role disappear and the two outer circuits synchronize with zero lag. This phenomenon, known as isochronous synchronization, holds when the relay circuit is replaced by a different dynamical unit; in this case, a Sprott circuit. In parallel experiments, not shown here, we have observed that the symmetry is the key ingredient of isochronous synchronization, and is lost when asymmetries are introduced in the coupling time or the coupling strength.

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APPENDIX A: THE CHUA CIRCUIT

Figure 6 shows a detailed description of the Chua circuit used in this work. A nonlinear resistor is connected to a set of passive electronic components (R,L,C). We have systematically studied the dynamical ranges of the circuit when R_{exc} is modified, observing stable, periodic, excitable, and chaotic dynamics. Among all of them, we drive the circuit to have

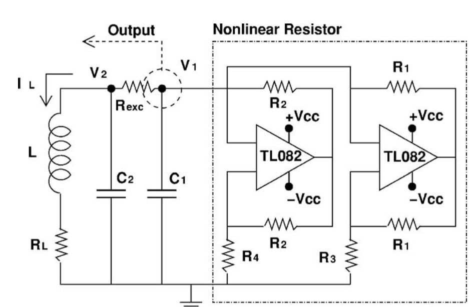

FIG. 6. Description of the Chua circuit, which is built with two TL082 operational amplifiers and passive electronic components of values V_{cc} =15 V, R_1=222 Ω, R_2=22 kΩ, R_3=2.2 kΩ, R_4=3.3 kΩ, R_L=23 Ω, C_1 =10 nF, C_2=100 nF, L=10 mH. We set R_{exc}=1.85 kΩ in order to have chaotic dynamics. V_1 and/or V_2 correspond to the outputs of the circuit, which are coupled to the other circuits through a voltage follower as shown in the experimental setup. Note that all the components have a 5% tolerance on their values.

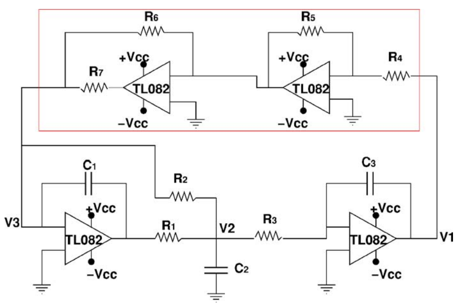

FIG. 7. (Color online) Description of the Sprott circuit. The circuit is composed of a linear integrator (lower part) and a nonlinear feedback loop (red square). The nonlinear function can be written as $f(x)=-A_3x+A_4\text{sign}(x)$. The numerical values of the components are $R_3=R_4=R_5=1\ \text{k}\Omega$, $R_1=220\ \Omega$, $R_2=1\ \text{M}\Omega$, $R_6=10\ \text{k}\Omega$, $R_7=31\ \text{k}\Omega$, $C_1=10\ \text{nF}$, $C_2=22\ \text{nF}$, $C_3=10\ \text{nF}$.

chaotic dynamics by setting $R_{\text{exc}}=1.73\ \text{k}\Omega$. Under these conditions, the dynamics of the circuit in the phase space given by (V_1,V_2) lies in a double-scroll chaotic attractor.[14] The output of the circuit (V_1 or V_2) is sent to the other circuits (with the same characteristics) via a voltage follower, in order to guarantee unidirectional injection (see Appendix C for details on the coupling implementation).

The dynamics of the circuit is described by the equations,[14]

$$C_1\frac{dV_1(t)}{dt}=\frac{V_2(t)-V_1(t)}{R_{\text{exc}}}-g(V_1,V_{cc}), \tag{A1}$$

$$C_2\frac{dV_2(t)}{dt}=\frac{V_1(t)-V_2(t)}{R_{\text{exc}}}+I_L+\frac{V_{\text{ext}}(t-\tau_c)-V_2(t)}{R_c}, \tag{A2}$$

$$L\frac{dI_L(t)}{dt}=-V_2(t)-R_LI_L(t), \tag{A3}$$

where the function $g(V_1,V_{cc})$ represents the characteristic curve of the nonlinear resistor, which is piecewise linear and contains a region of negative resistance. The last term of Eq. (A2) accounts for the external coupling, which depends on the voltage of the coupled circuit V_{ext} (introduced after a delay τ_c) and the coupling strength, regulated by the coupling resistance R_c.

APPENDIX B: THE SPROTT CIRCUIT

The second chaotic circuit, named after J. C. Sprott,[23] is a simple circuit composed of three linear integrators with a nonlinear feedback loop. We adjust the parameters of the circuit to show a chaotic double scroll structure. The schematic representation is shown in Fig. 7. The circuit simulates a third-degree differential equation, called a "jerk." The equations modeling the circuit are

$$\frac{V_1(t)}{dt}=V_2(t)+\frac{V_{\text{ext}}(t-\tau_c)-V_2(t)}{R_c}, \tag{B1}$$

$$\frac{V_2(t)}{dt}=V_3(t), \tag{B2}$$

$$\frac{V_3(t)}{dt}=-A_1V_3(t)-A_2V_2(t)-A_3V_1(t)+A_4\text{sign}[V1(t)], \tag{B3}$$

where the last term of Eq. (B1) accounts for the external coupling. The coefficients A_1, A_2, A_3, and A_4 depend on the circuit parameters in the following way:

$$A_1=\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right),$$

$$A_2=\frac{1}{R_1C_2R_2C_1},$$

$$A_3=\frac{1}{R_6C_1R_1C_2R_3C_3},$$

$$A_4=\frac{-V_0}{C_1R_7R_1C_2R_3C_3}.$$

The numerical values of the components of the circuit are shown in the caption of Fig. 7.

APPENDIX C: COUPLING AND DELAY

The coupling between each circuit is introduced by a digital delay line, which samples and buffers the signal before restoring it τ_c ms later. The circuits are coupled with different input/output variables, depending on the direction of the signal. V_1 is the output variable, whereas the input signal is injected into variable V_2. This mechanism of asymmetric coupling prevents feedback loops in the circuits, and therefore the signals are completely decoupled. The Sprott circuit is coupled in a similar way, the variable V_1 is sent to the other circuits while the variable V_2 receives the incoming signals through the coupling resistance. Details of the coupling scheme are shown in Fig. 8. Each signal is buffered with an "op-amp" in order to preserve the dynamics of the circuit.

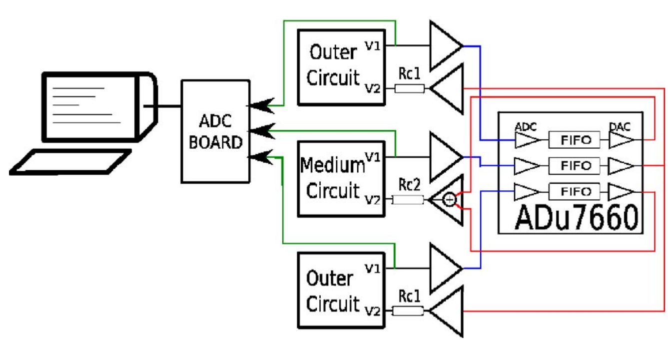

FIG. 8. (Color online) Experimental setup used for the coupling of the circuits. The output voltage V_1 of each signal is sampled in the microcontroller ADu7660 and then buffered into a digital memory before being returned to the circuits. Each triangle represents a signal conditioner in order to adapt the signal to the specification of the ADC inputs. The blue line represents the injected signal and the red lines are the delayed signals reintroduced into the circuit. The central circuit receives the sum of the signals of the outer circuits. Each signal is reintroduced into variable V_2 through a resistance R_c. Note that this resistance might be different for the central circuit.

The experimental setup is made up of several blocks. The first part consists of the chaotic circuit. Each circuit is connected to the digital delay line and to the Analog-to-Digital Converter (ADC) acquisition board (see blue lines in Fig. 8). The digital delay line is composed of an autonomous microcontroller with on-board memory and a Digital-to-Analog Converter (DAC) and ADC. The microcontroller is an ADu7660 development board from Analog Devices. The signal is first converted to digital signal and then stored into a FIFO (i.e., First In First Out) buffer in order to introduce the delay. After a number of clock ticks, the signal is then converted into analog. These converters sample signals up to 50 Khz with 12-bit precision and the delay can be chosen up to $128/f_e$, with f_e being the chosen sampling frequency. In the experiments the sampling frequency of the microcontroller is chosen to be f_e=50 KHz.

The analog signals from the circuits are then sampled with an ADC sampling board connected to a computer and the signals are later analyzed with Matlab software. The variable V_1 of each circuit is sampled at ten times their mean frequency, that is, above 40 Khz, and with a precision of 12 bits.

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