

Controlling crisis-induced intermittency using its relation with a boundary crisis

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Abstract. In this paper, we show that there is a simple and intuitive relation between crisis-induced intermittency and a boundary crisis in unimodal maps and in a family of chaotic flows with strong volume contraction. This point of view allows us to identify a set of ‘target points’ directly from the time series of the considered system that can be used to tame crisis-induced intermittency. To do this, we use an advantageous adaptation of a control scheme that was typically used in the context of control of transient chaos. We illustrate here these ideas with the quadratic map and Rössler flow.

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1. Introduction

The control of chaotic systems [1] has applications in a wide variety of fields in science [2, 3]. The aim of any control scheme is to obtain desired dynamical behaviour from a system by applying small but accurately chosen perturbations. This goal can usually be achieved by

making a wise use of the properties of the dynamical system considered. As a paradigmatic example, Ott *et al* [4], in their pioneering work, showed how to stabilize the system in any of the unstable periodic orbits embedded in a chaotic attractor by applying small perturbations to one of the system parameters, making use of the system ergodicity and of the saddle-type instability of these periodic orbits. Another well-known control method, the method of Pyragas [5, 6], is also based on the existence of those periodic orbits. It has also been shown that chaotic behaviour can be suppressed by simply applying small harmonic perturbations destroying the chaotic attractor, and finally leading the system to periodic behaviour [7, 8].

Nevertheless, permanent chaos is not the only type of chaotic behaviour that appears in nature, so that other control schemes have been designed that are adapted to other dynamical situations. For example, many works have dealt with the control of transient chaos [9]–[12], where the goal is to keep the system close to the unstable chaotic set (and thus far from an undesired attractor). In this context, the problem of chaos preservation in situations where a bifurcation leads to the destruction of the chaotic attractor has also drawn a good deal of attention [13, 14].

However, less attention has been paid to finding accurate ways of controlling an important phenomenon in dynamical systems: crisis-induced intermittency [15]. This phenomenon takes place after an interior crisis when, by varying one of the system parameters, the chaotic attractor of a dynamical system experiences a sudden expansion after touching one of the unstable orbits lying in its basin of attraction. After an interior crisis, a trajectory typically alternates periods of time in the region where the pre-crisis attractor lay with excursions out of it. Different techniques to control this phenomenon have already been proposed [16]–[18]. The practical importance of these control methods derives from the fact that they allow one to keep the trajectories of a chaotic attractor bounded in certain regions of phase space, something that might be desirable in some contexts. For example, in [17, 18] by controlling crisis-induced intermittency in a CO₂ laser the trajectories are kept in a zone of phase space where the pre-crisis attractor lay, and for this physical system this implies that the laser intensity is drastically reduced, something that has important technical implications.

The mechanism of an interior crisis is quite well understood, and for unimodal maps it is properly explained in [15]. However, in [18] an interior crisis is related to a boundary crisis, where a chaotic attractor is destroyed giving rise to a non-attractive chaotic set (a chaotic saddle), in a simple and intuitive way. In this paper, we are going to describe this relation in further depth. Furthermore, using the Rössler system as an example, we show that this relation can also be extended to the family of chaotic flows for which the return maps built by plotting two consecutive local maxima (or minima) of a given variable of the system lie on an approximate one-dimensional map function. Different examples of this type of flow can be found in [11, 19].

Here, we also describe a potential application of this relation. We show that it can be used to design a control scheme to suppress crisis-induced intermittency. We propose a method to achieve this goal based on the existence of certain ‘target points’ in phase space, which turns out to be a variation of a control technique previously used to keep the trajectories of a system that has experienced a boundary crisis close to the chaotic saddle [11]. We show that our novel point of view allows one to identify those target points directly from a *return map* of the time series of the considered system, without having previous knowledge of the system equations. This is the main and most important difference between the scheme that we propose here and those proposed in previous references [16]–[18]. In this paper, we also show that by using this

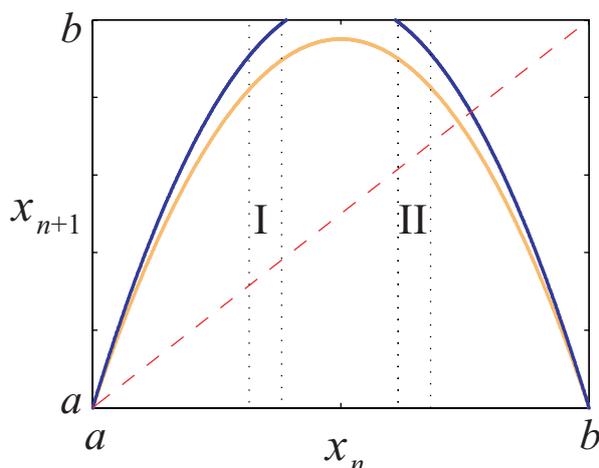


Figure 1. Plots of a trapping map (orange) and a non-trapping map (blue) in the interval $L = [a, b]$. The dotted lines show the bounds of intervals I and II, which play an important role in our control strategy. Note that the points in the interval between I and II are mapped out of L after one iteration of the non-trapping map. The line $x_{n+1} = x_n$ is plotted to illustrate the difference between these two maps.

control method we can keep the trajectories of a chaotic system that has experienced an interior crisis in the region where the pre-crisis attractor lay with small and infrequent perturbations (limited in practical situations).

The structure of the paper is as follows. In section 2 we show the relation that exists between boundary and interior crisis using a one-dimensional map, and then we extend this idea to chaotic flows using the Rössler system as a paradigmatic example. After this, in section 3 we show how our point of view can help to identify certain target points that are used to tame crisis-induced intermittency in the chaotic systems considered. Finally, in section 4 we draw the main conclusions of this paper.

2. Interior crisis as a small-scale boundary crisis

We can now address how, for some chaotic systems, interior crisis can be understood in terms of a small-scale boundary crisis taking place in the chaotic attractor. To illustrate this idea, we need some previous definitions. Consider first a one-dimensional unimodal map of the form

$$x_{n+1} = f(x_n, C), \quad (1)$$

like the one shown in figure 1. We say that the map is *trapping* on an interval $L = [a, b] \subset \mathbb{R}$ if $f(L) \subset L$. A *non-trapping* map will be a map such that $L \subset f(L)$ and such that for almost all initial conditions $x_0 \in L$, except for a zero measure set, there is a natural number n for which $f^n(x_0) \notin L$. It is clear that an orbit of a trapping map starting in L will stay in L forever. However, for a non-trapping map, typically an orbit starting in L will escape from L after a certain number of iterations, after which it might come back or not to L , depending on the form of the map f out of L . Figure 1 illustrates the difference between these two types of maps. We can see clearly how the points in the L interval are mapped inside L by a trapping map (orange (grey)). Nevertheless, for a non-trapping map (blue (black)), we can see in the figure that there

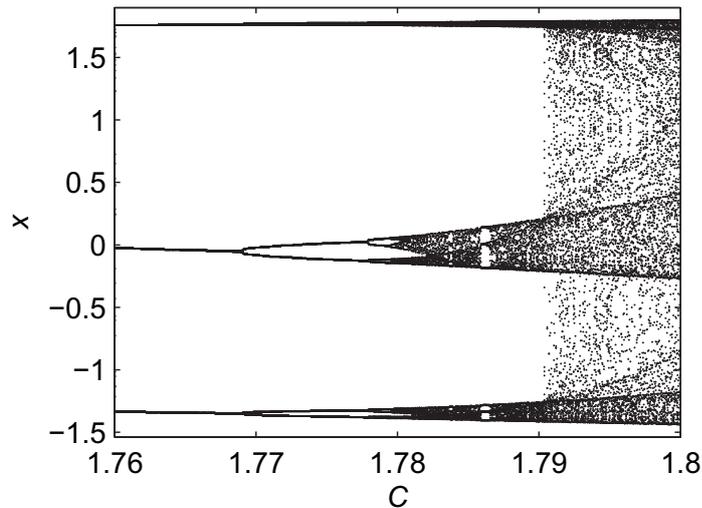


Figure 2. Bifurcation diagram for the quadratic map $x_{n+1} = C - x_n^2 = F(C, x_n)$. For values of C slightly smaller than the value $C^* \approx 1.79$, trajectories are confined in three intervals. At $C = C^*$ the interior crisis occurs, so for $C > C^*$ trajectories alternate fractions of time in these three intervals with excursions out of them, a phenomenon known as crisis-induced intermittency.

is an interval between two small intervals I and II (which will later play an important role in our control strategy) such that points inside it are mapped out of L after one iteration.

With these basic notions in mind, consider now that for $C \leq C^*$ the map of (1) is trapping, and that for $C > C^*$ it becomes non-trapping. If the trajectories that fall out of the interval L never come back to it, obviously the attractor that might have existed inside L for $C \leq C^*$ is destroyed for $C > C^*$. The transition that takes place at $C = C^*$ is known as a boundary crisis [15].

We can now focus on an interior crisis, and see how it is related to this phenomenon. To do this, we consider the quadratic map

$$x_{n+1} = C - x_n^2 = F(C, x_n), \quad (2)$$

which is conjugate to the well-known logistic map [20], and for which the phenomenon of the interior crisis has been extensively studied [15, 21, 22]. For this system, at $C = C_s \approx 1.76$ a stable period-three orbit is born, a phenomenon referred to as subduction. If we increase the parameter C , there is a period doubling bifurcation cascade giving rise to an attractor whose points are contained in three intervals but, after a critical value, $C^* \approx 1.79$, the attractor touches the unstable period-three orbit that lies in its basin of attraction. As a consequence, a one-piece chaotic attractor is obtained, where trajectories alternate periods of time inside intervals of the pre-crisis attractor with excursions out of these intervals. This alternate behaviour is precisely the crisis-induced intermittency. This process is summarized in the bifurcation diagram shown in figure 2.

We call L_1 , L_2 and L_3 the three intervals in which the chaotic attractor is bounded for $C \approx 1.78 < C^*$, as we can see in figure 3. The key idea here is to see how the map $F^3(C, x) \equiv F(C, F(C, F(C, x)))$ behaves on each of the intervals L_1 , L_2 and L_3 . We can see in figure 3(b) that for this value of C the map $F^3(C, x)$ is trapping on L_2 , which implies that it

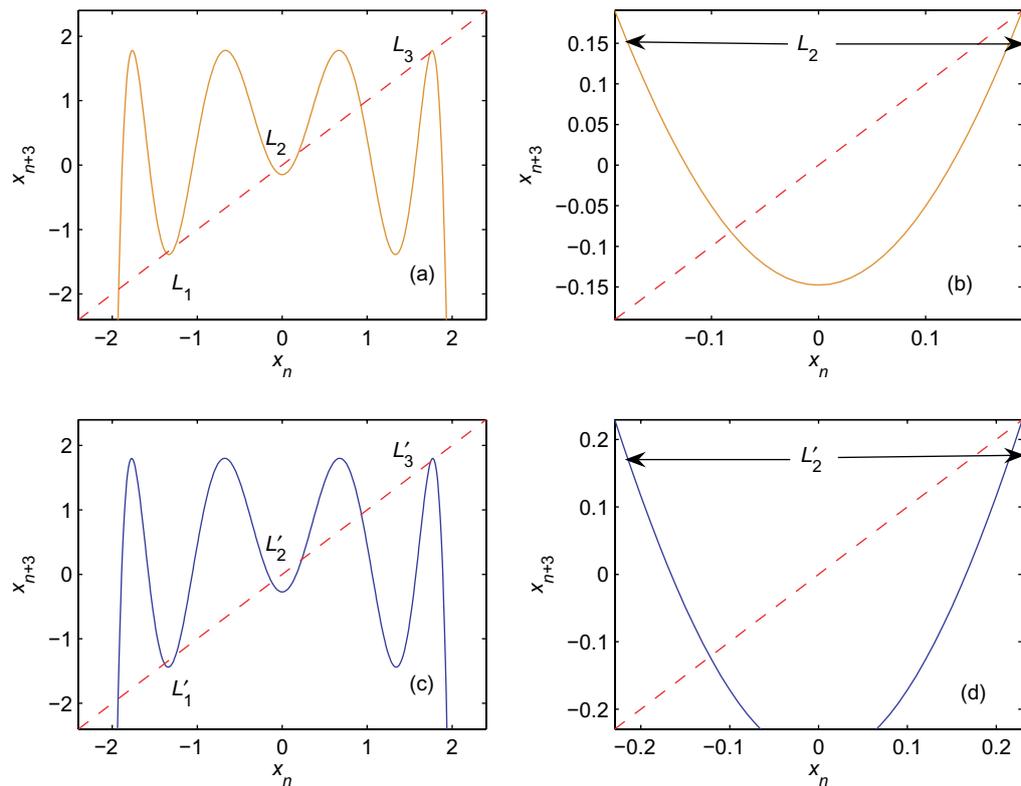


Figure 3. (a) A graph of x_{n+3} against x_n for $C < C^*$ (orange), before the crisis, showing the approximate position of intervals L_1 , L_2 and L_3 . (b) A zoom of the same graph showing that $F^3(C, x)$ is trapping on L_2 , which implies that the trajectories are confined inside these three intervals. (c) A graph of x_{n+3} against x_n for $C > C^*$ (blue) after the crisis, showing the approximate position of intervals L'_1 , L'_2 and L'_3 . (d) A zoom of the graph close to L'_2 showing that the map $F^3(C, x)$ is non-trapping on this interval, which gives rise to the post-crisis attractor covering the interval and crisis-induced intermittency.

is also trapping on L_1 and L_3 , and this is the reason why trajectories are confined inside these intervals. However, if we take a value of the parameter C past the critical value $C > C^*$, we can find three new intervals L'_1 , L'_2 and L'_3 , close to the intervals L_1 , L_2 and L_3 , such that $F^3(C, x)$ is not a trapping map for those intervals. This is shown in figure 3(d) for L'_2 , and an analogous graph can be obtained for L'_1 and L'_3 . This is the reason why we say that an interior crisis can be understood as a small-scale boundary crisis. A key difference with this phenomenon, though, is that for $C > C^*$, trajectories that fall out of L'_1 , L'_2 and L'_3 under $F^3(C, x)$ can come back to them after a number of iterations. As mentioned before, this is the main feature of crisis-induced intermittency.

The fact mentioned above that for $C > C^*$ the map $F^3(C, x)$ becomes non-trapping simultaneously for L'_1 , L'_2 and L'_3 is a consequence of how the map $F(C, x)$ acts. If for example $F^3(C, x)$ becomes non-trapping in L'_1 , it means that $L'_1 \subset F^3(C, L'_1)$. If we apply the map $F(C, x)$ to this relation, we have $F(C, L'_1) \subset F^3(F(C, L'_1))$. And if we apply again the map $F(C, x)$ to this relation, we have $F^2(C, L'_1) \subset F^3(F^2(C, L'_1))$. These relations imply that

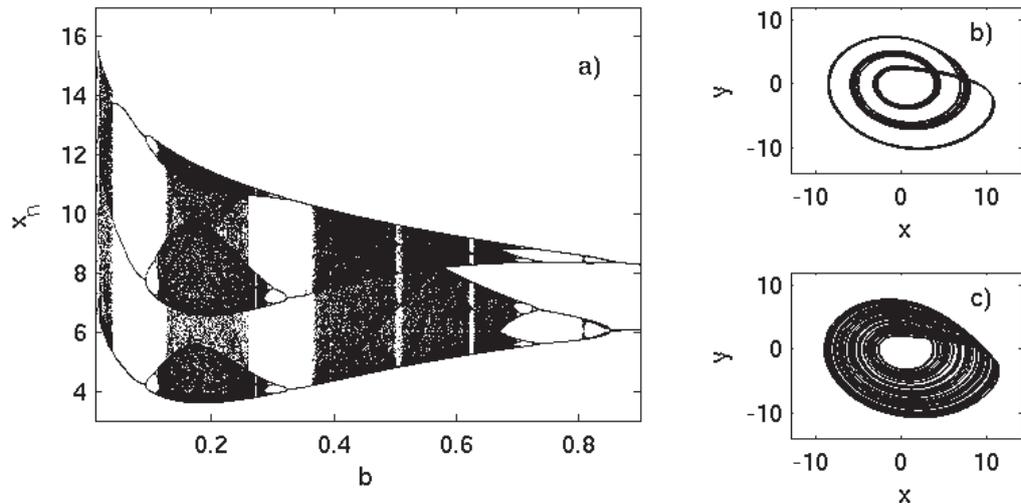


Figure 4. (a) Bifurcation diagram for the Rössler system as a function of parameter b . An interior crisis takes place for $b^* \approx 0.26$. (b) Sparse chaotic attractor for $b = 0.27 > b^*$, before the crisis. (c) Filled-in chaotic attractor arising after the interior crisis, for $b = 0.2 < b^*$.

$L'_2 \subset F^3(C, L'_2)$ and $L'_3 \subset F^3(C, L'_3)$, i.e. the map is non-trapping in either L'_2 or L'_3 , where we identify $L'_2 \equiv F(C, L'_1)$ and $L'_3 \equiv F(C, L'_2)$.

The point of view adopted above can also be applied to the analysis of interior crisis for chaotic flows, provided that the dynamics of the flow considered can be somehow related to that of a one-dimensional map. This turns out to be a frequent situation. Since the pioneering work of Lorenz [23], it is known that sometimes, by representing the $(n+1)$ st local maximum (minimum) against the (n) th local maximum (minimum) of a variable of the flow considered, i.e. by making a *return map*, the points seem to lie in an approximate one-dimensional map function. To our knowledge, no theory exists by which we can know beforehand that the dynamics of a flow can be related to that of a one-dimensional map in this way. However, the common feature of flows for which this relation holds [19] is that they are typically highly contractive and the dynamics become highly elongated along a one-dimensional unstable dimension, in such a way that the remaining directions (transverse to it) become difficult to discern. Examples of chaotic flows for which this relation applies, which are used to model different systems of interest in physics, chemistry, engineering and biology, can be found in [11, 19].

A paradigmatic example of a chaotic flow of this type is the Rössler system, so we can use it to illustrate how our point of view can be extended to the dynamics of this type of chaotic flow. The equations of this system are

$$\dot{x} = -y - z, \quad \dot{y} = x + ay, \quad \dot{z} = b + z(x - c). \quad (3)$$

We show in figure 4(a) the bifurcation diagram of the Rössler system as a function of parameter b , considering $a = 0.2$ and $c = 5.7$. In particular, the consecutive local maximum values of the variable $x(t)$, denoted x_n , are represented against b . By decreasing the value of this parameter, we can observe how the system undergoes some interior crises. In order to illustrate our point of view, we pay attention to the one occurring at $b^* \approx 0.26$, where a sudden expansion of the chaotic attractor occurs, i.e. from a sparse chaotic attractor where x_n is contained in three

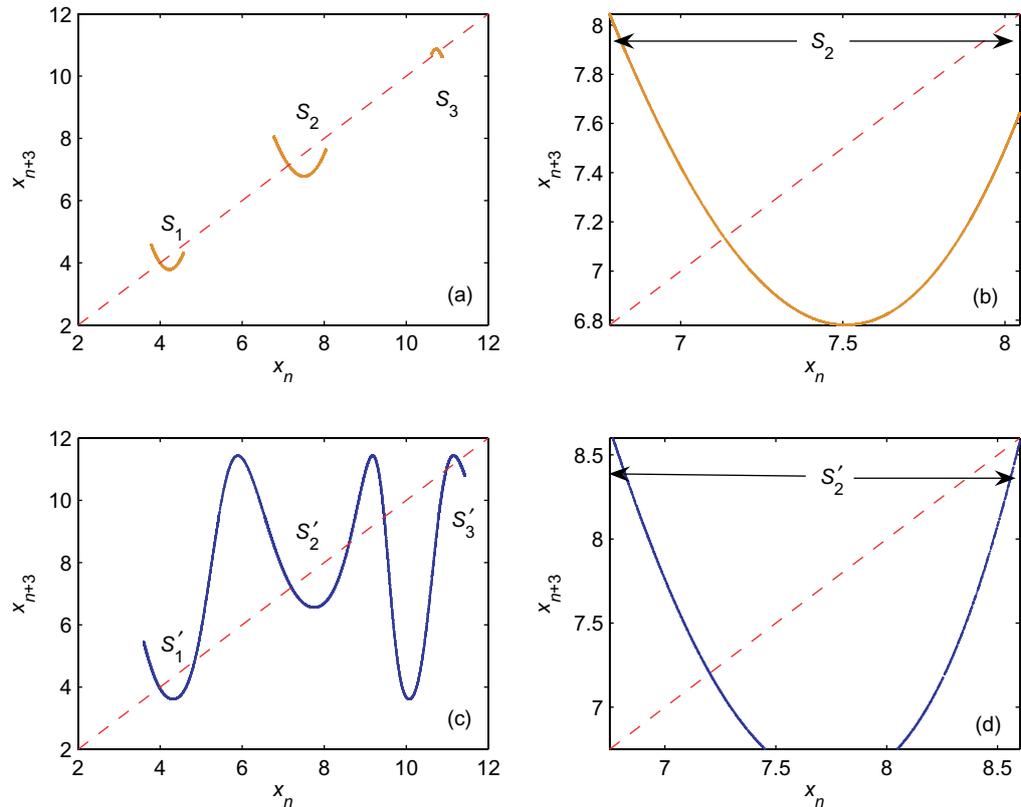


Figure 5. (a) A graph of x_{n+3} against x_n for $b > b^*$ (orange), before the crisis, showing the approximate position of intervals S_1 , S_2 and S_3 , where x_n is the n th local maximum of the variable $x(t)$ of the Rössler system. (b) A zoom of the same graph showing that $G^3(b, x)$ is trapping on S_2 , which implies the existence of a sparse attractor for this value of b . (c) A graph of x_{n+3} against x_n for $b < b^*$ (blue), after the crisis, showing the approximate position of intervals S'_1 , S'_2 and S'_3 . (d) A zoom of the graph close to S'_2 , showing that the map $G^3(C, x)$ is non-trapping on this interval, which gives rise to the post-crisis filled-in attractor.

different intervals, shown in figure 4(b), to a filled-in chaotic attractor where x_n moves on a large one-piece interval, shown in figure 4(c).

The procedure to extend our point of view of interior crisis as a small-scale boundary crisis to this chaotic flow is the following: for this system, if we represent x_{n+1} against x_n , points lie on an approximate one-dimensional map function, which we call $x_{n+1} = G(x_n, b)$. For $b \approx 0.27 > b^*$, we can see in figure 5 that the Rössler system has a sparse chaotic attractor, where x_n is contained in three different intervals S_1 , S_2 and S_3 . We can represent the map $G^3(b, x_n) : \mathbb{R} \rightarrow \mathbb{R}$ against x_n simply by representing x_{n+3} against x_n . This map would take the value of any local maximum of the variable x , that is, x_n , and map it into the third posterior local maximum, that is, x_{n+3} . The key observation, as before, is that the numerically calculated map $G^3(b, x_n)$ is a trapping map on each of those intervals S_1 , S_2 and S_3 , and that is the reason why the chaotic attractor presents this sparse appearance, which can be clearly seen in figure 4(b). However, if we take a value of the parameter slightly smaller than b^* , such as for example, $b = 0.2$, we can find three new intervals S'_1 , S'_2 and S'_3 close to the former ones such that

$G^3(b, x_n)$ is a non-trapping map for those intervals, as for the quadratic map. The identification of these intervals can be accurately done guided by the eye, provided that we have a sufficiently good approximation of the curve $G(b, x_n)$. To do this, we have to note that each interval S'_i should be close to the pre-crisis interval S_i and that it is bounded by points x_a^* and x_b^* such that $x_a^* = G^3(x_a^*, b)$, and $x_b^* = G^3(x_b^*, b)$. The consequence of this change into a non-trapping map is that for $b < b^*$ we have a filled-in attractor as shown in figure 4(c). This idea is illustrated in figure 5.

This point of view can be extended to the family of maps and chaotic flows specified above. Now we are going to show that it can be applied to control crisis-induced intermittency.

3. Application to control intermittency

In this section, we show that the connection revealed in section 2 between interior crisis and boundary crisis can be used to tame crisis-induced intermittency.

This can be done by adapting a control scheme that was previously used to control trajectories of a system that has suffered a boundary crisis [11]. Consider a non-trapping map such as the one shown in figure 1. If we build the set of points that do not escape from the interval L under iterations of the map, this set will be similar to a Cantor set. Using this idea, it is easy to see that in the intervals I or II there are points that come back to either I or II after a large number of iterations, which we might call ‘target points’, that can be eventually detected using time series. Thus, if we take one of them as an initial condition, we just need to wait until the trajectory comes back to I or II and to perturb it $x \rightarrow x + \Delta x$ to make it lie again on the closest target point, and this can be repeated forever, so that the controlled trajectory remains inside L . Note that the smaller the I and II, the smaller the correction Δx ; hence for this control scheme the upper bound to the correction that needs to be applied to the trajectory is determined by the size of I and II. The main difference between this algorithm and the one proposed in [11] is that in the latter the target points chosen inside I and II are those that stay in L a long time before diverging. Thus, the idea there is to apply a correction each time a trajectory falls in the interval between I and II, where all trajectories fall before diverging, and steer the trajectory to the closest of those target points. However, it is clear that the correction applied using this strategy will be bounded by half the length of the escaping interval. In this way, the perturbations in our method turn out to be smaller than in the method proposed in [11], although typically we would need a bigger number of time series of the system (or a bigger computational effort) to reduce the frequency of the control perturbations.

On the other hand, it is clear that in our strategy intervals I and II could be set in ways different from the one shown in figure 1. However, setting them as proposed above has an advantage: if the system is eventually affected by noise or if there is a control error, trajectories will fall in the interval between I and II before diverging, so we can always apply a correction to place it back on a target point on I and II, which is minimal with this setting.

Now we explain how this technique can be specifically used to control crisis-induced intermittency. As we said, for $C > C^*$, by plotting $x_{n+3} = F^3(C, x_n)$ against x_n we can find three intervals L'_1 , L'_2 and L'_3 such that $F^3(C, x)$ is a non-trapping map in those intervals. For any of those intervals (for example L'_2), we can use the control technique described above to keep the trajectories inside this interval under iterations of $F^3(C, x_n)$ and thus suppress crisis-induced intermittency. It is important to note that our point of view allows one to identify the subintervals I and II and to detect the target points by using time series of the considered system,

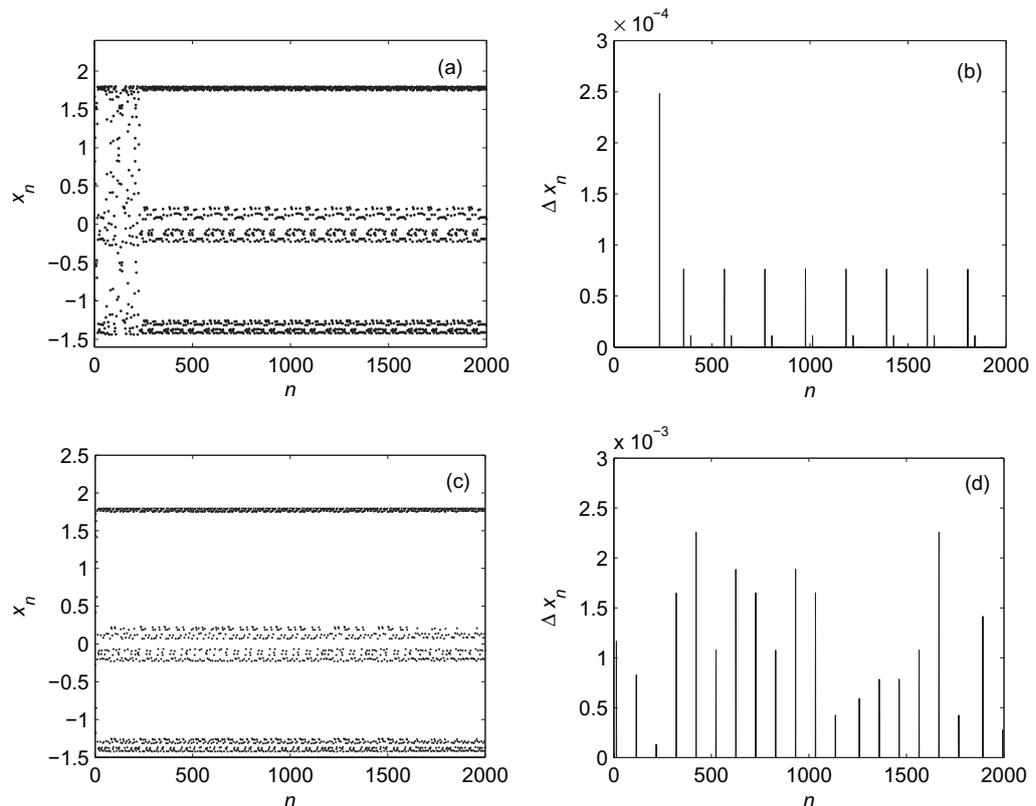


Figure 6. (a) A controlled trajectory obtained by placing the trajectory on the closest target point each time it falls on I or II. Note that the points of the trajectory (dotted) lie in three intervals as before the interior crisis. (b) The control applied to the trajectories, Δx , and its dependence on time. We can see how this control is small and infrequent. (c) A controlled trajectory obtained by placing the trajectory on a random target point each time it falls on I or II. (d) The applied control, which is small and infrequent again.

so in principle no additional information is needed. An important advantage when applying our method in this context is that, contrary to what happens when applying it to control a system that has suffered a boundary crisis, any trajectory can be confined to the zone where the pre-crisis attractor lay, provided that the trajectories of the post-crisis attractor will pass close to I and II sometime (the smaller the intervals, the bigger the time we will typically have to wait), and once they fall there we can start applying the control method.

As a numerical example of application, we have used this idea to keep trajectories of the quadratic map bounded in the pre-crisis region when initially considering $C = 1.8 > C^*$. In the simulations shown, we chose interval I and II of length 0.005 and we computed 20 target points just by iterating trajectories whose initial conditions lay in I or II, in such a way that an initial condition was chosen as a target point if it came back to either I or II after a sufficiently large number of iterations (here, 25). The results are shown in figure 6. We can see in figure 6(a) that the trajectories are kept inside three intervals, as before the interior crisis, and it is clear from figure 6(b) that the control applied is small and infrequent.

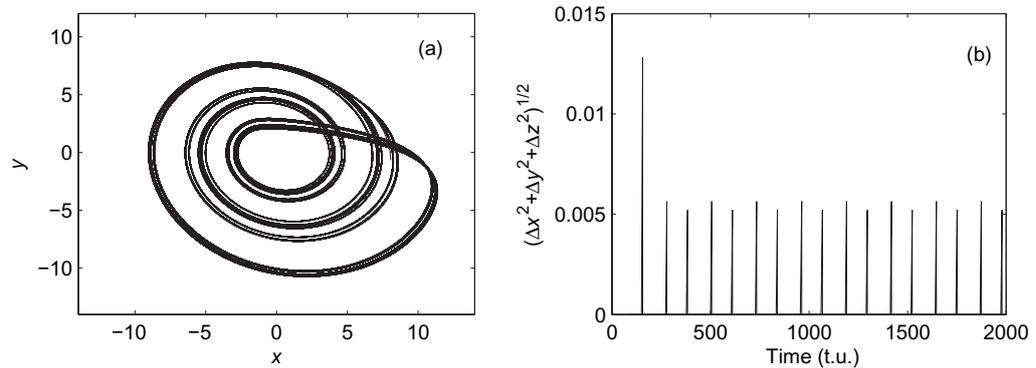


Figure 7. (a) A controlled sparse chaotic attractor for $b = 0.2$. The appearance is the same as before the interior crisis. (b) The control applied to the trajectories and its dependence on time. We can see how this control is quite small and infrequent.

Simulations show that the application of our method leads typically to a periodic orbit of high period. However, we might be interested in having a controlled attractor that looks chaotic as the pre-crisis attractor. In that case, there is a simple way of overcoming this difficulty. Instead of placing the trajectory on the closest target point each time it falls into I or II, we just have to place it on a randomly chosen target point on I or II, so that the resulting orbit will not be periodic. This can be observed in figures 6(c) and (d).

Considering the ideas given in section 2, an analogous strategy can be applied to tame intermittency for chaotic flows. We can briefly explain it using the Rössler system. As mentioned before, by plotting $x_{n+3} = G^3(b, x_n)$ against x_n we can find three intervals S'_1 , S'_2 and S'_3 such that $G^3(b, x_n)$ is a non-trapping map in those intervals. Again, intervals I and II and the target points can be found using time series. The only difference is that we will have to keep record of the x , y and z coordinates of the target points, and apply a correction to the three coordinates $x \rightarrow x + \Delta x$, $y \rightarrow y + \Delta y$ and $z \rightarrow z + \Delta z$ each time x_n falls in either I or II. We expect the corrections applied to be of the order of the length of I and II.

A numerical application of our control technique for $b = 0.2 < b^*$ is shown in figure 7. For this example, we used 151 target points that lay in intervals I and II of length 0.1 arbitrary units. We can see in figure 7(a) that the controlled system shows all the time the sparse chaotic appearance that it presented before the interior crisis, contrary to the appearance of the uncontrolled attractor that can be observed in the inset plot of figure 5. On the other hand, it is clear from figure 7(b) that the applied perturbations are infrequent, and their amplitude $(\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$ is typically small. It is important to emphasize again that our method does not require one to know the explicit equations of the system of interest; the return map and the observation of the system trajectories are enough in order to identify the target points. In this case, by reconstructing the map of x_{n+3} against x_n with a relatively short time series, we detected the intervals I and II where the target points should lie. We have integrated the system for a long time, and each time a local maximum fell in I or II the point (with its three coordinates) was stored as a 'target point' if the trajectory remained confined and came back to I or II after a sufficiently long time.

We want to point out that our technique allows one to keep trajectories of the considered system in the region where different pre-crisis attractors lay, and thus to transform a filled-in

attractor into different sparse attractors. We have observed that for the examples considered, and for different values of C , by representing the graph of $F^k(C, x)$ (where $F(C, x)$ is either a map or the return map of a flow) against x it is possible to find k subintervals L_1, \dots, L_k such that the map $F^k(C, x)$ is non-trapping on them. This is typically because the system presents an interior crisis for a value C_k^* close to C , in such a way that, for example, for $C < C_k^*$ the variable x is confined in k subintervals and for $C > C_k^*$ we have crisis-induced intermittency. Thus, for values of $C \gtrsim C_k^*$, the trajectories of the system can be controlled using our procedure so the resulting controlled trajectory mimics the trajectories of the sparse chaotic attractor observed before such an interior crisis.

Finally, we want to remark that the basic idea of finding a family of target points such that orbits starting on them spend a long time in the region where the pre-crisis attractor lay before they escape can also be applied to tame crisis-induced intermittency for dynamical systems with low dissipation rates. However, considering that these systems will not have a nearly one-dimensional return map, we might need some more information to determine in an accurate way the regions in phase space (analogous to our intervals I and II) where those new ‘target points’ might be found.

4. Conclusions

In this paper, we have given an alternative point of view for the analysis of the phenomenon of interior crisis and a method to control the dynamics arising after this phenomenon: crisis-induced intermittency. We have shown that for one-dimensional maps, the phenomenon of interior crisis can be understood as a small-scale boundary crisis. Furthermore, we have shown that for the class of chaotic flows that can be related to this type of map, an analogous point of view can be adopted to analyse the phenomenon of crisis-induced intermittency. With these ideas in mind, we have shown that there is a way to keep the trajectories of an attractor that has suffered an interior crisis inside the zone where the pre-crisis attractor lay, which is based on techniques used to control trajectories of systems that have suffered a boundary crisis, and for which the target points needed can be readily identified from time series using our point of view. This method requires small and infrequent corrections on the system trajectories, although the minimum perturbation and frequency that is necessary is determined by practical limitations. We have also argued that our method allows one to confine the trajectories of a chaotic attractor in the region of phase space where different pre-crisis attractors remained bounded.

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