



Energy Harvesting Enhancement by Vibrational Resonance

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The idea to use environmental energy to power electronic portable devices is becoming very popular in recent years. In fact, the possibility of not relying only on batteries can provide devices longer operating periods in a fully sustainable way. Vibrational kinetic energy is a reliable and widespread environmental energy, that makes it a suitable energy source to exploit. In this paper, we study the electrical response of a bistable system, by using a double-well Duffing oscillator, connected to a circuit through piezoceramic elements and driven by both a low (LF) and a high frequency (HF) forcing, where the HF forcing is the environmental vibration, while the LF is controlled by us. The response amplitude at low-frequency increases, reaches a maximum and then decreases for a certain range of HF forcing. This phenomenon is called vibrational resonance. Finally, we demonstrate that by enhancing the oscillations we can harvest more electric energy. It is important to take into account that by doing so with a forcing induced by us, the amplification effect is highly controllable and easily reproducible.

Keywords: Energy harvesting; vibrational resonance.

1. Introduction

In the last few years, a quick development has occurred in the miniaturization capability of electronic devices. On the other hand, the same improvement speed has not been comparable for the energy density available in batteries that provide the power for such devices, when operating in stand-alone configurations [Paradiso & Starner, 2005]. Thus, the possibility to overcome the limitations related to the power requirement of small electronic components has become an important

research field. One recent idea is to power such small electronic devices by using energy available in their environment. This is the core of so-called *energy harvesting*. The main goal that energy harvesting aims to achieve is to reduce the requirement of an external source as well as the maintenance costs for periodic battery replacement and the chemical waste of conventional batteries. Due to its diffusion, an interesting possibility has received growing attention, i.e. converting the micro-kinetic energy, mostly available as random motion often

manifested as *vibration*, into electrical power. This is called *kinetic energy harvesting* [Kazmierski & Beeby, 2011]. Basically, three mechanisms to convert vibration to electric energy have been proposed, electromagnetic [Williams & Yates, 1996], electrostatic [Mitcheson *et al.*, 2004] and piezoelectric [Litak *et al.*, 2010] transduction. We have focused our attention on the piezoelectric mechanism because it shows some advantages such as the large power densities and the ease of application [Cook-Chennault *et al.*, 2008], among others. Most piezoelectric energy harvesters are in the form of a cantilever beam with one or two piezoceramic layers. The beam is located on a vibrating host structure and the dynamic strain induced in the piezoceramic layers gives an alternating voltage output. Recently, a larger interest has been focused on beam-mass systems [Erturk & Inman, 2011; Tang *et al.*, 2010]. A new model of an inverted beam coupled to a piezoelectric transducer has been proposed [Friswell *et al.*, 2012]. This inverted beam has a tip mass making the vertical position unstable, so that the beam buckles, giving rise to a double-well potential due to the gravitational loading. Another bistable oscillator was proposed by [Moon & Holmes, 1979], but it was only considered for energy harvesting 30 years later [Erturk *et al.*, 2009]. This harvester is composed of a ferromagnetic beam with a patch of piezoelectric material and two external magnets. In this case, the bistability is given by the higher magnetic permeability of the ferroelectric material. In this paper, we study the possibility to enhance the output electrical signal of a ferromagnetic harvester by using a resonant phenomenon that has been studied for the first time by Landa and McClintock [2000]. The authors of the previous paper have shown that in a bistable system driven by both a low and a high frequency forcing, the response amplitude at the low frequency increases, reaches a maximum and then decreases as the amplitude of the high frequency forcing is varied. This phenomenon is called *vibrational resonance* (VR). So far, this phenomenon has been thoroughly studied in a large class of dynamical systems such as a noise induced structure [Zaikin *et al.*, 2002], coupled oscillators [Gandhimathi *et al.*, 2006], biological oscillators [Daza *et al.*, 2013], multistable systems [Jayakumari *et al.*, 2009], spatially periodic potential systems [Rajasekar *et al.*, 2011], among others. In particular, in [Rajasekar *et al.*, 2011], the authors demonstrate theoretically and

numerically that a biharmonic signal can give rise to VR in periodic potential systems, as the oscillator is driven by the biharmonic force $F \cos \omega t + f \cos \Omega t$, when F, f are the amplitudes of the excitations and $\Omega \gg \omega$ are the frequencies. The main idea of this paper is to model the environmental vibration as a periodic HF forcing then to use controllable LF forcing in order to enhance the amplitude of the strain of the beam when the VR phenomenon appears. It is useful to remember that the amplitude of the oscillations of the beam and the output electrical signal are directly related, through the piezoelectric transduction mechanism. It is important to notice that as the LF forcing can be controlled by us, the enhancement of the electrical output is highly controllable. To conclude, we study the enhancement of the electrical power generated by a cantilever beam in the presence of VR.

The organization of the paper is as follows. In Sec. 2, we study the model and the nature of the system. Section 3 is a description of the usual treatment of VR in dynamical systems and how we apply it to our model. In Sec. 4, we examine the results of our simulations. Finally, a discussion and the main conclusions of this paper are summarized in Sec. 5.

2. Model Description

In order to study VR in an energy harvesting system, we have chosen a model based on the double-well Duffing oscillator, that is shown in Fig. 1, and whose equations are:

$$\begin{aligned} \ddot{x} + 2r\dot{x} - \alpha x(1 - x^2) - \chi v \\ = A \cos(\omega t) + B \cos(\Omega t) \end{aligned} \quad (1)$$

$$\dot{v} + \lambda v + \kappa \dot{x} = 0. \quad (2)$$

The variable x is the dimensionless transverse displacement of a beam tip, r is the damping ratio, v is the dimensionless voltage across the load resistor, χ is the dimensionless piezoelectric coupling term in the mechanical equation, κ is the dimensionless piezoelectric coupling term in the electrical equation, $\lambda \propto 1/RC$ is the reciprocal of the dimensionless time constant of the electrical circuit, R is the load resistance, and C is the capacitance of the piezoelectric material. Next, α is the stiffness parameter, and A, B are, respectively, the low and high frequency inertial forces and $\omega \ll \Omega$ the respective frequencies. The amplitudes of the two

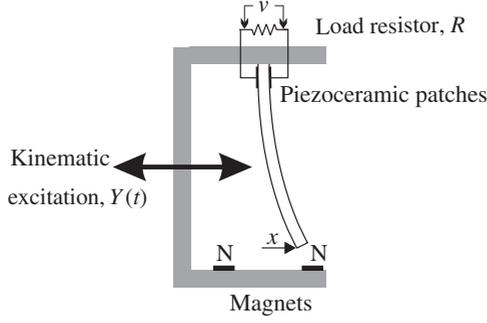


Fig. 1. Schematic plot of the harvesting device consisting of the ferromagnetic beam, the magnets, the piezoelectric patches and the electrical circuit.

kinematic forcing are functions of the respective frequencies $A = F\omega^2$, $B = f\Omega^2$ because the periodic forcing terms are related with the simple harmonic movement. All the values of the parameters that we have used in the simulations are described in Table 1.

A plot of the potential of our system, $V(x) = -\frac{x^2}{4} + \frac{x^4}{8}$, the double-well Duffing oscillator, is shown in Fig. 2. The stable equilibrium points of our system are $(1, 0)$, $(-1, 0)$ and the unstable equilibrium point is $(0, 0)$, which corresponds with the vertical position of the beam. Before to continue describing the system it should be useful to remember that in Eq. (1), we have two different forcings, $A \cos(\omega t)$, $B \cos(\Omega t)$ and two different frequencies, Ω and ω . From now on, we take $\Omega = 9$ and $\omega = 0.5$. Depending on the initial conditions, all the trajectories inside the two-well potential fall in one of the equilibrium points in the absence of forcing. Eventually, if the amount of forcing is large enough, the trajectories could be wider and never fall into the well. This is important because in our case the wider the trajectories the bigger the harvested energy as

Table 1. The simulation parameters.

Parameters	Description	Value
r	Dissipation	0.5
χ	Dimensionless piezoelectric coupling term in the mechanical equation	0.05
κ	Dimensionless piezoelectric coupling term in the electrical equation	0.5
λ	Reciprocal of the dimensionless time constant	0.01
α	Stiffness mistuning parameter	0.5

Note: The table shows the parameters that we have used in the simulations and their values.

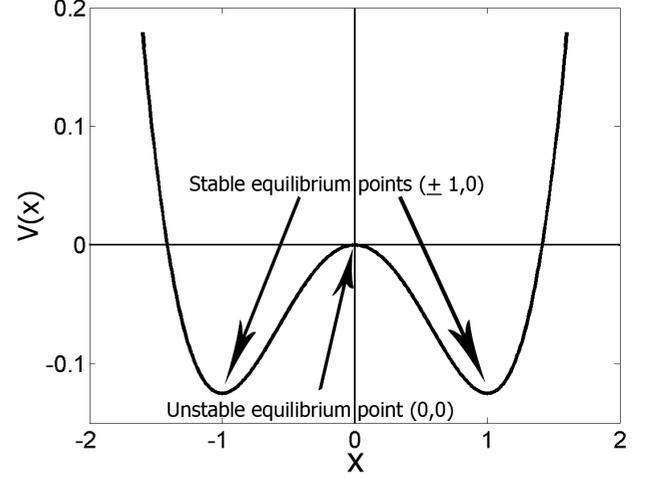


Fig. 2. The figure plots the potential of the system. Black arrows denote the positions of the equilibrium points.

we can see in Figs. 3(a)–3(d). In fact, in these figures we show the displacement along the x axis and the power v^2 in a case with $A = 0$ [Figs. 3(a) and 3(b)], and with $A = 0.7$ [Figs. 3(c) and 3(d)]. It is possible to see that the difference between the power generated is strongly related with the amplitude of the displacement. For the simulations, we have discarded in both cases the transients and have used an external forcing $f = B/\Omega^2 = 1$. Here and from now on, we start every simulation from the initial condition $(x, y, v) = (1, 0, 0)$.

3. Vibrational Resonance in the Energy Harvesting System

Vibrational resonance consists of the optimization of the response of the system to a low-frequency (LF) signal of amplitude A and frequency ω due to a high-frequency (HF) signal of amplitude B and frequency $\Omega \gg \omega$. Our present purpose is to study numerically the phenomenon of VR for the variable x , and the power v^2 . The system of differential equations that we have to solve is defined by Eqs. (1) and (2). The relation between the output and the forcing signals provides an idea of how the LF signal is being amplified by the HF signal. This is commonly defined by means of the Q factor, which is defined as:

$$Q = \frac{\sqrt{(C_s^2 + C_c^2)}}{A}. \quad (3)$$

Here A is the LF forcing amplitude and Q is the response for the frequency ω , usually defined as the amplitude of the sine and cosine components

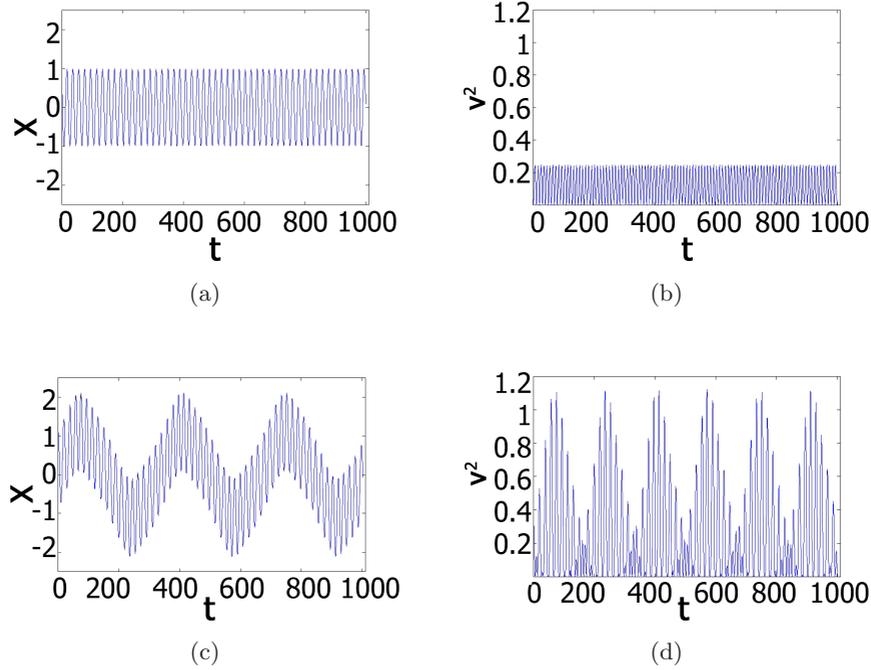


Fig. 3. Figures (a) and (b) show respectively the displacement of the beam along the x -axis and the power v^2 with $F = A/\omega^2 = 0$. Figures (c) and (d) show respectively the displacement of the beam along the x -axis and the power v^2 with $A = F\omega^2 = 0.7$, amount of forcing that produces VR. In both cases, we have discarded the transients and used an external forcing $f = B/\Omega^2 = 1$. The initial condition that we use in Eqs. (1) and (2) is $(x, y, v) = (1, 0, 0)$.

of the output signal, yielding

$$C_s = \frac{2}{nT} \int_0^{nT} \Gamma(t) \sin \omega t dt \quad (4)$$

$$C_c = \frac{2}{nT} \int_0^{nT} \Gamma(t) \cos \omega t dt. \quad (5)$$

The function $\Gamma(t)$ used to calculate the Q factor is, in our case, $x(t)$ or $v^2(t)$, n is the number of complete oscillations of the LF signal and $T = 2\pi/\omega$ is its period. The numerical values of C_s and C_c are related to the Fourier spectrum of the time series of the variable v^2 computed at the frequency ω . The usual procedure to search for VR is to compute Q for different amplitudes B of the HF periodic signal [Landa & McClintock, 2000]. If there is a value of B that maximizes Q , then the VR phenomenon occurs. This means that there is a particular value of the HF periodic signal that optimizes the response of the system to the weak LF periodic signal.

4. Numerical Results

Here, in order to analyze the VR phenomenon in our system we solve the differential equations [Eqs. (1)

and (2)] in the absence of the LF controlled external signals and then apply both the LF and HF periodic signals once the transient has vanished. Then, we compute the factor Q for a range of different values of the HF amplitude $B = f\Omega^2$ and for different fixed LF amplitude $A = F\omega^2$. Now a graph of Q_x , Q_{v^2} and Q_v versus $f = B/\Omega^2$ is plotted and, if the parameters are properly chosen, the typical bell-shaped curve is found [Figs. 4(a), 4(b) and 5(a)]. The maxima of these curves are the optimal match between the LF and HF signals and appear when the VR phenomenon occurs. Note that Q_x shows double peaks for fairly small values of A , corresponding to the single and double-well oscillation response. In contrast, Q_{v^2} shows only a single peak informing that the output power oscillations in the system with a single well resonating response are relatively small. After having analyzed the Q factor results, we discuss the influence of the VR on the power output. Figure 5(a) shows the corresponding Q_v factor which mirrors the double peaks behavior from Q_x as shown in Fig. 4(a). This is due to the linear piezoelectric coupling in Eqs. (1) and (2). Furthermore, the average power output is plotted in terms of the voltage variance $\sigma^2(v)$ in Fig. 5(b). Note that the maximum is reached for a

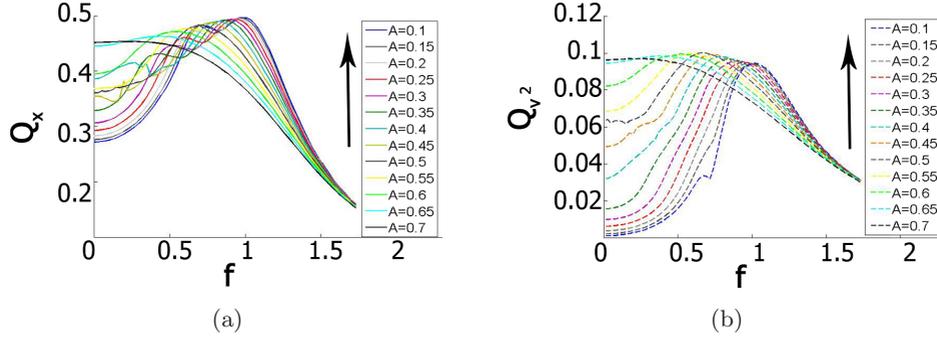


Fig. 4. Figures (a) and (b) plot respectively the Q factor calculated for x and v^2 versus $f = B/\Omega^2$ for different values of $A = F\omega^2$. $A = 0.1, A = 0.15, A = 0.2, A = 0.25, A = 0.3, A = 0.35, A = 0.4, A = 0.45, A = 0.5, A = 0.55, A = 0.6, A = 0.65, A = 0.7$. The initial condition that we use in Eqs. (1) and (2) is $(x, y, v) = (1, 0, 0)$. The black arrows in the figures indicate the increasing direction of parameter A along the different curves.

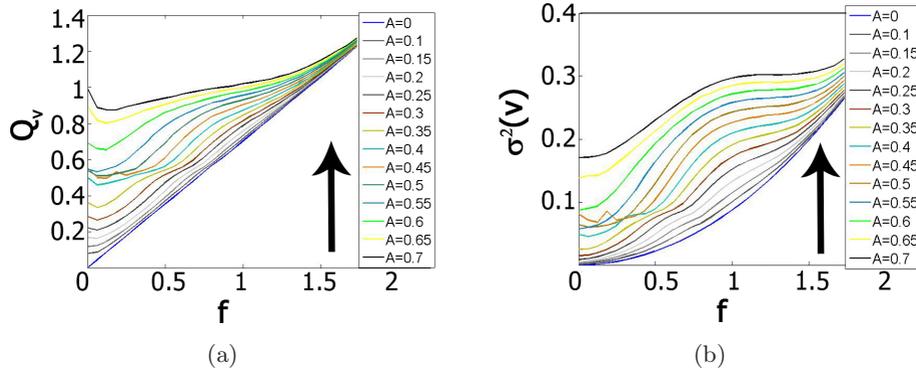


Fig. 5. Figures (a) and (b) plot respectively Q_v and $\sigma^2(v)$ versus $f = B/\Omega^2$ (the parameter values A , the initial conditions and the black arrow as in Fig. 4).

large value of f , but for intermediate values of A , we observe a certain increase of the power output in the region of the first peak of VR. This corresponds to the synchronization of the optimal output displacement just before the bifurcation from single to double-well oscillations. For better clarity, we have estimated the standard deviation of

the beam displacement $\sigma(x)$ and power generated $\sigma(v^2)$ in the same range of kinematic forcing amplitude f as shown in Figs. 4(a) and 4(b). In Figs. 6(a) and 6(b), it is possible to compare the increments of the beam displacement and the power response, respectively. These figures confirm our previous conclusions on VR on the two output parameters. This

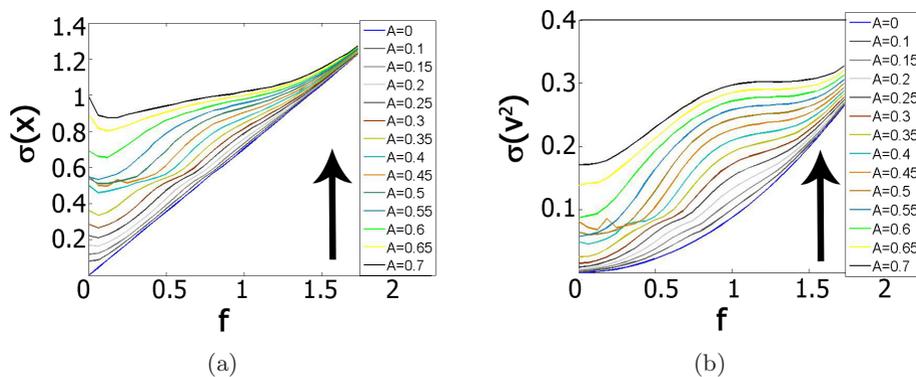


Fig. 6. Figures (a) and (b) plot respectively the standard deviations $\sigma(X)$ and $\sigma(v^2)$ versus $f = B/\Omega^2$ (the parameter values A , the initial conditions and the black arrow as in Fig. 4).

result is strongly related with the Q curves (Fig. 4), where the maxima appear at the same amount of f inside the bell curves. Furthermore, the displacement of the beam along the x axis and the power v^2 for forcing values of A and f that produce the VR phenomenon appear in Figs. 3(c) and 3(d). To summarize, we have studied that it is possible to find a periodic perturbation controlled by us that can enhance the environmental vibration effect in a double-well oscillator. Moreover, the increase in oscillations' amplitude gives us the possibility to produce more energy through the piezoelectric patches stuck on the cantilever beam.

5. Conclusions

We have analyzed the relationship between the VR and the power response v^2 of a double-well Duffing oscillator, designed to harvest energy. We have used two periodic forcings in the Duffing oscillator, one with high frequency Ω , the environmental vibration, while the other has a low frequency ω , the induced vibration. Our study has permitted to find values of the low frequency forcing $f = B/\Omega^2$ for which the VR is verified. As we expected, the power gained is strongly related with the amplitude of the oscillations. Therefore, in these range of values we have calculated the power response and the displacement of the system for different high frequency forcing amplitudes A and we have put them in relation with the power response for $A = 0$. We have shown that when the VR appears, the displacement $x(t)$ expands, so that the power $v^2(t)$ increases. It is important to underline that as the VR phenomenon is induced by a controllable forcing applied to the system, the power enhancement is easily reproducible and controllable. Finally, we discuss that both increments of $x(t)$ and $v^2(t)$ follow the bell shaped curve of the Q factor. To conclude, the vibrational resonance can produce positive effects on the power produced by a harvesting energy device from the environmental kinetic energy.

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