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Research paper

Recovering an unknown signal completely submerged in strong noise by a new stochastic resonance method

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ABSTRACT

Unknown signal recovery plays always a crucial role in the discipline of signal processing. Especially, a signal completely submerged by a strong noise is more difficult to be restored and identified in the engineering fields. Here, we provide an effective method to recognize the types and related parameters of an unknown signal in a strong noise background. Firstly, the nonlinear vibration approach is adopted to enhance an unknown weak signal with the assistance of proper noise, in which a new quantitative indicator is designed to keep the resonance response to follow the unknown signal features. Subsequently, the polynomial fitting and the variance of the time difference sequence are implemented to estimate several important signal parameters. Finally, the frequency spectrum of the recovered signal is compared with that of the original signal to verify the correctness of the restored signal. Recovery results of three typical signals indicate that the proposed method is effective. Moreover, unknown weak signals are obviously enhanced and signal features are completely preserved. The proposed method successfully takes advantage of the energy of the complex noise components. This work may pave the way for recovering unknown signal from a strong noise background.

1. Introduction

Signals are usually considered as the characterizations of running states for the equipment and system subjected to the change of working conditions. Typically, these signals are obtained in the engineering practice by means of sensors. Hence, numerous noise components existing in the target signal inevitably results in the difficulty of the signal feature identification. It is always a valuable study to recover an unknown weak signal from the noise background, in particular, when a signal is totally contaminated by a strong noise.

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To carry out the signal recovery, some different and effective methods are taken into account. In the traditional signal recovery approaches, the method of compressive sensing [1] treated as an important technique has attracted much attention, since the sparse signal is able to be captured from the measurements [2]. With the help of the compressive sensing theory, some effective approaches, such as the orthogonal matching pursuit [3,4], random projections [5,6], and the computationally convex optimization program [7], have been developed. The sparse signal needs to satisfy the linear measurements in the compressive sensing system [8]. Thus, the problem of signal recovery has been further studied under the conditions of incomplete and inaccurate measurements [9,10]. In addition to these conventional methods, some novel techniques of signal recovery have been also reported. The deep learning approach [11], fractional Fourier transform [12], discrete-time wavelet transform maxima [13], and sparse Bayesian learning [14] have been also implemented in the signal recovery. In general, the noise involved in the aforementioned approaches is merely Gaussian white noise. Recovering a sparse signal from the exponential-family noise has been investigated [15]. From the perspective of signal components, vibration signals collected from the rotary machines are more complex and more difficult to be recovered. To solve this problem, the methods of the super-exponential algorithm [16], adaptive filtering [17] and spectral kurtosis and ensemble empirical mode decomposition [18] have been employed to recover defective vibration signals from a noise background.

All above-mentioned approaches applied to signal recovery systems have obtained satisfactory results. However, the algorithms of these methods are complex and inefficient. Especially, for the case of a weak signal completely submerged by strong stochastic components, these methods may be invalid for clearly recovering the original weak signal. In addition, most of the methods are to highlight the signal by eliminating the noise, which may also weaken the target signal. Hence, it is necessary to enhance an unknown weak signal before recovering it to make sure the characteristics of the unknown signal more clearly. As we know, nonlinear systems have a good amplification performance for a weak signal by exploiting the noise energy [19,20]. Therefore, nonlinear systems are extraordinarily suitable systems to process weak signals with a large amount of noise.

Considering the advantages of nonlinear systems, we use here a bistable nonlinear system as a processing system to enhance and recover an unknown weak signal [21]. A new indicator is designed to quantify the response of nonlinear systems. Three signals with different characteristics are adopted to demonstrate the effectiveness and correctness of a recovery method by the analysis of the time-frequency domain. All results indicate that the three signals are exactly restored and some important parameters are estimated. This work may be helpful for the signal recovery in the wide range of fields including circuit systems [22], biological systems [23], and mechanical systems [24], among others.

2. Bistable nonlinear system

The process of signal recovery is sketched in Fig. 1. Gaussian white noise is chosen as a random excitation to simulate a complex noise background. A weak signal contaminated by a strong noise background has structured the state of an unknown signal. In such a situation, the type and parameters of the signal are completely unknown. Therefore, it is necessary to select an appropriate method for the recognition of an unknown weak signal.

For a stochastic excitation, nonlinear systems have a good performance for transforming noise energy to enhance the weak signals. In this paper, we choose a classical bistable system as a recovery system. Bistable systems have wide applications in the fields of physics, biology and other disciplines [22–24]. Furthermore, the research findings based on a bistable nonlinear system are easily extended to multi-stable and more complex systems. In bistable dynamics, a weak periodic perturbation brings about a modulation of the two-well potential separated by the potential barrier and introduces a periodical change for a potential well switching. The state transition of the bistable system modulated by a weak periodic signal is described in Fig. 2. In the process of periodic switching between two potential wells, there is a critical amplitude in the system [25]. When the amplitude of the periodic disturbance is the maximum or minimum, the critical amplitude means that the modulated bistable system changes from the bistable state to a monostable state. When the conversion rate switched from one potential well to the other in a double-well potential is twice the frequency of the periodic perturbation, It occurs that the overall synchronous switching of the bistable dynamics. Under this circumstance, a weak signal is enhanced owing to the resonance of a nonlinear system. The conversion rate depends only on the amount of noise when the parameters of the bistable system are constant. Hence, the existence of an optimal noise intensity is to achieve the best synchronous switching, and then to induce the optimal resonance response. An unknown weak narrowband periodic perturbation is obviously amplified. This phenomenon is described as stochastic resonance (SR) [26–28]. The SR can be mathematically depicted by the following governing equation in a bistable system subjected to the excitation of the signal with a strong noise under the overdamped condition [21,27],

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -U'(x) + S(t) + N(t), \tag{1}$$

where $S(t)$ and $N(t)$ are the unknown signal and random perturbation, respectively. The random perturbation is defined as $N(t) = \sqrt{2D}\xi(t)$, where D denotes the noise intensity, and $\xi(t)$ is the standard Gaussian white noise with zero mean and Brownian variance. The potential function $U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$ denotes a symmetric double-well potential, where $a > 0$ and $b>0$ are the characteristic parameters of the system, which are usually lied in scope [0, 2]. Eq. (1) essentially describes the overdamped motion of a Brownian particle in a double-well potential driven by the external periodic force and noise.

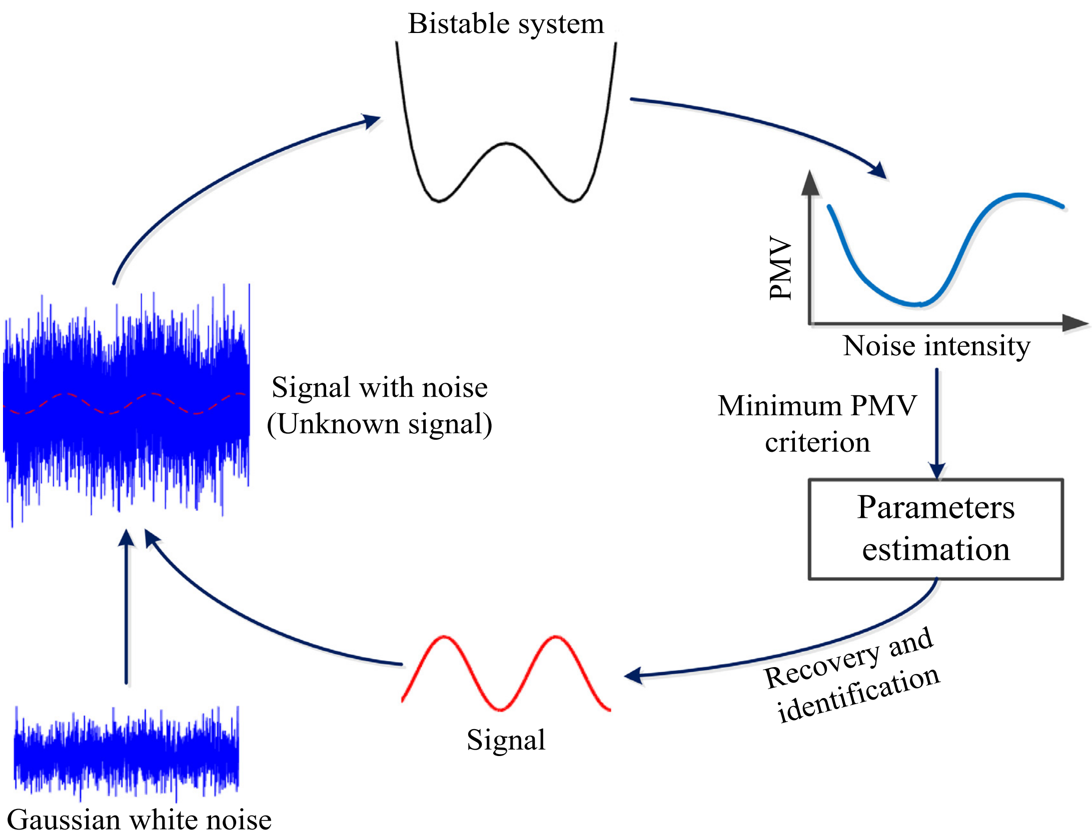

Fig. 1. The process of recovering an unknown signal. The target signal is assumed as a completely unknown signal, which is buried by a strong Gaussian white noise. The signal (red dotted line) with noise is regarded as a stochastic excitation of the bistable system to induce SR. Then, an unknown weak signal is enhanced by exploiting noise energy in the nonlinear system evaluated by the new PMV indicator. A series of noise intensities are tested to find out the minimum PMV value characterizing the optimal resonance of nonlinear systems. According to the optimal resonance response, several key parameters are estimated to identify the type of signal. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

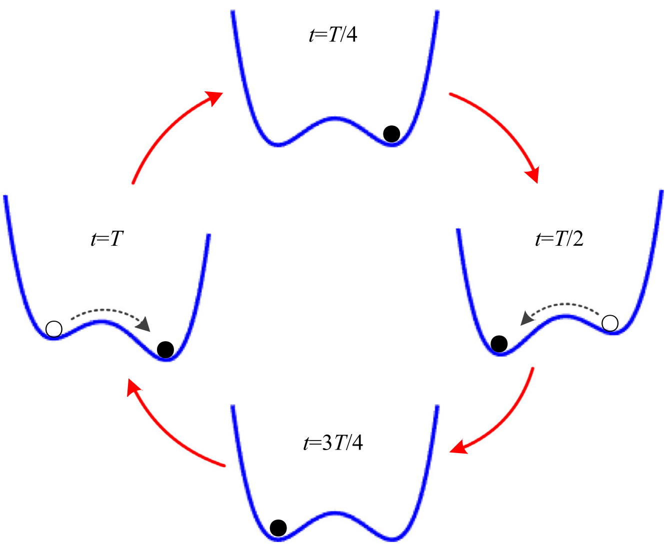

Fig. 2. The cyclic transition of the bistable system modulated by a weak periodic signal. T is the cycle of the signal.

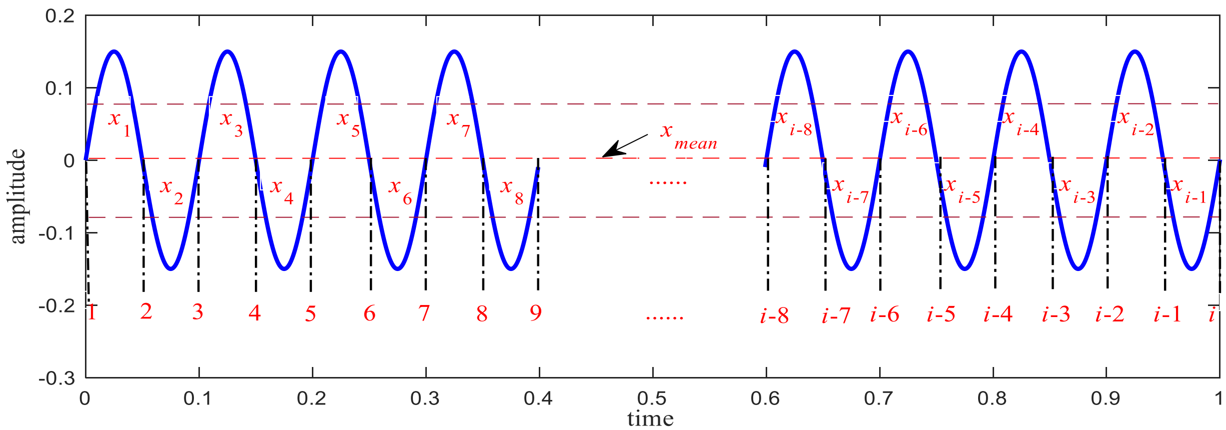

Fig. 3. The realization of PMV indicator by using the harmonic signal as an example.

The potential barrier $\Delta V=\frac{a^2}{4b}$ separates the two equilibrium points $x_{1,2}=\pm\sqrt{\frac{a}{b}}$. They are located in a positive region and a negative region, separately. Consequently, the resonance response from the double-well potential is in full-wave form.

The unknown signal is improved by the SR system quantified by the piecewise mean value (PMV) indicator. A sequence of the noise intensities is implemented to search for the minimum PMV value. We perceive that the enhancement effect of the unknown signal is optimal when the value of PMV is minimum. A simple strategy is designed to estimate several critical parameters, and to verify the consistency of the recovered signal with the original signal.

3. Piecewise mean value indicator

Here we explain in detail the PMV indicator and the strategy of parameters estimation mentioned in Fig. 1. When we attempt to enhance a weak signal by a nonlinear system, some conventional indices are used to evaluate the output performance of the nonlinear systems on the basis of frequency domain, such as signal-to-noise ratio [29], and spectral power amplification [30]. However, the specific signal frequency needs to be known earlier than using these indicators. Hence, they cannot do anything to help the extraction of features of the unknown periodic signal and aperiodic signal. On the other hand, to overcome the limitation of frequency-domain indicators, time-domain-based correlated kurtosis index [31], weighted-kurtosis index [25], and impulse index and crest index [32], have been developed to assess the nonlinear response. These time-domain indicators are sensitive to the pulse components contained in a target signal. For target signals without the pulse components, the sensitivity and quantitative performance of these indicators may be extremely poor. Accordingly, they can only estimate the existence of pulse components in a target signal. Additionally, the output signal characteristics of nonlinear systems evaluated by the aforementioned time-domain indices cannot well follow the characteristics of the unknown input signal.

In view of the aforementioned limitations of the traditional indices, we will establish a new time-domain indicator dubbed PMV to qualify the performance of the nonlinear system response. Specifically, the realization steps of the new PMV indicator are as follows.

1) Counting all zero crossing points of the time-domain spectrum obtained from the nonlinear system response, namely, all points intersecting the time-domain waveform with the zero horizontal line. The length of the discrete time sequence is M, which is considered as an excitation of nonlinear system. The number of the zero crossing points may be i $(0<i<M)$;
2) The output time-domain spectrum of the nonlinear systems is divided into i-1 sub-segments with all zero crossing points i as the piecewise points;
3) Calculating the mean amplitude of the time-domain spectrum in all sub-segments, which is marked as x_k ($k=1, 2, 3, \ldots, i$-1);
4) Calculating the mean amplitude of the whole time-domain spectrum, which is marked as X_{mean};
5) Calculating the difference values between x_k and X_{mean} and averaging them within the whole range.

Therefore, the mathematical expression of the PMV indicator is concluded as shown in Eq. (2),

$$PMV=\frac{1}{i-1}\sum_{k=1}^{i-1}(x_k-X_{mean}). \tag{2}$$

According to the realization procedures of the PMV indicator, we know that the value of the PMV indicator is close to zero for the signals with a central horizontal line (full-wave signals). Taking the sinusoidal signal as an example, the process of realizing PMV indicator is shown in Fig. 3. It is easy to find that the PMV value of a sinusoidal signal is zero. The smaller the PMV value is, the smaller the difference of the mean amplitudes of all sub-segments is, and the closer the signal is to the full-wave signal.

Eq. (1) describes the fundamental theory of SR. SR has a special mechanism that converts abundant noise energy at the high-frequency region to the feature frequency of the target signal to enhance signal characteristics [25,26]. When the Brownian particle is in one of the potential wells and the signal amplitude is far less than the critical amplitude [25], the Brownian particle can only stay at the bottom of a potential well, and oscillates in a smaller range with the change of the potential well. However, the presence of noise changes the situation. With the help of noise, Brownian particle accumulates enough energy to jump the potential barrier. This process will consume a period. For signals with varying amplitudes, the smaller the particle amplitude, the more time is required to accumulate energy. The Brownian particle with sufficient energy moves from the original potential well to the other potential well, which produces the enhancement of the nonlinear system output. Therefore, the output time-domain spectrum distributes on both sides of the zero horizontal line at the better resonance conditions, which constitutes a full-wave output signal with the bipolarity. The output of nonlinear systems keeps the similar characteristics with the raw signal, simultaneously. The minimum PMV value is further obtained corresponding to the optimal SR response. For this case, the characteristics of the output signals are highly consistent with those of the unknown input signals. To a certain extent, the PMV indicator reflects the dispersion degree of the nonlinear system response amplitude. Hence, the minimum PMV value shows the optimal resonance state.

4. Parameters estimation strategy

We can ascertain the types of unknown signals through the nonlinear system and the PMV indicator. However, the signal type is not the complete evidence to verify the signal features. Therefore, a simple strategy of the parameter estimation is exhibited to confirm the specific signal. First, after polynomial fitting for the output waveform of nonlinear systems, the sampling time points corresponding to the zero crossing points of the fitting curve are calculated. The time difference values between the adjacent zero crossing points are obtained. The values of the time difference can respond to the periodic information of an unknown signal to a certain extent. Further, we can compute the variance of the time difference sequence, which explains the discreteness of the time difference values. Next, we divide the time difference sequence into three types in accordance with the magnitude of the variance:

(i) when the variance is extremely close to zero, the discretization of the time difference sequence is smaller and the time difference values are approximately equal. Consequently, we consider the unknown signal as a periodic signal. Combining with the shape of fitting curve, if the fitting curve is in smooth harmonic form, it can be determined that the unknown signal is a harmonic signal. If there are some small cosine waves in the pulse width of the fitting curve, the unknown signal can be regarded as a square wave signal. Because square wave signal is formed by superposition of a series of harmonic components, harmonic signal can be roughly viewed as a special case of square wave signal. Therefore, it can be determined that the unknown signal is a square wave signal according to some small cosine fluctuations in the pulse width of a periodic signal. In view of a periodic signal, the period of the unknown signal is the mean of the time difference sequence, which reflects the frequency of an unknown signal;

(ii) when the variance is far larger than zero, it indicates that there is a larger discreteness existing in the time difference sequence. Combining with the shape of the fitting curve, it can be viewed as an aperiodic signal;

(iii) when the variance is slightly larger than zero, the signal type can be preliminarily recognized by combining the fitting curve and the time difference curve. Then, we calculate the reciprocals of the time difference values and perform polynomial fitting to them. Thus, the frequency variation trend of the unknown signal is identified according to the polynomial coefficients.

Finally, the spectrum diagram of a recovered signal is compared with that of the original signal to approve the correctness of the signal recovery method on account of the nonlinear system and the PMV indicator. The specific procedure of realizing the PMV indicator and parameters estimation strategy is shown in Fig. 4.

5. Unknown signal recovery verification

The aforementioned introduction and analysis have presented the recovery procedure of an unknown signal. Harmonic, binary and chirp signals, which denote three typical full-wave signals with different characters, are adopted to demonstrate the reliability of the unknown signal recovery method. The expression forms of different signals are as follows,

$$\begin{gathered} S_H(t) = A_H \sin(2\pi f_H t) \\ S_B(t) = A_B \sum_{j=-\infty}^{\infty} S_j \Gamma(t - jT),\ \Gamma(\bullet) = \begin{cases} 1, \bullet \in [0, Q] \\ 0, others, \end{cases}, \\ S_C(t) = A_C \cos\left(\pi \beta t^2 + 2\pi f_C t + \psi\right) \end{gathered} \tag{3}$$

where $S_H(t)$, $S_B(t)$ and $S_C(t)$ are harmonic signal, binary signal, and chirp signal, respectively. The signal amplitudes are $A_H = 0.5$, $A_B = 0.5$, and $A_C = 0.6$, respectively. The frequency of the harmonic signal is $f_H = 100$ Hz, and the initial frequency of the chirp signal is $f_C = 1$ Hz. For the high-frequency signals, the classic SR method is invalid. In this section, we adopt

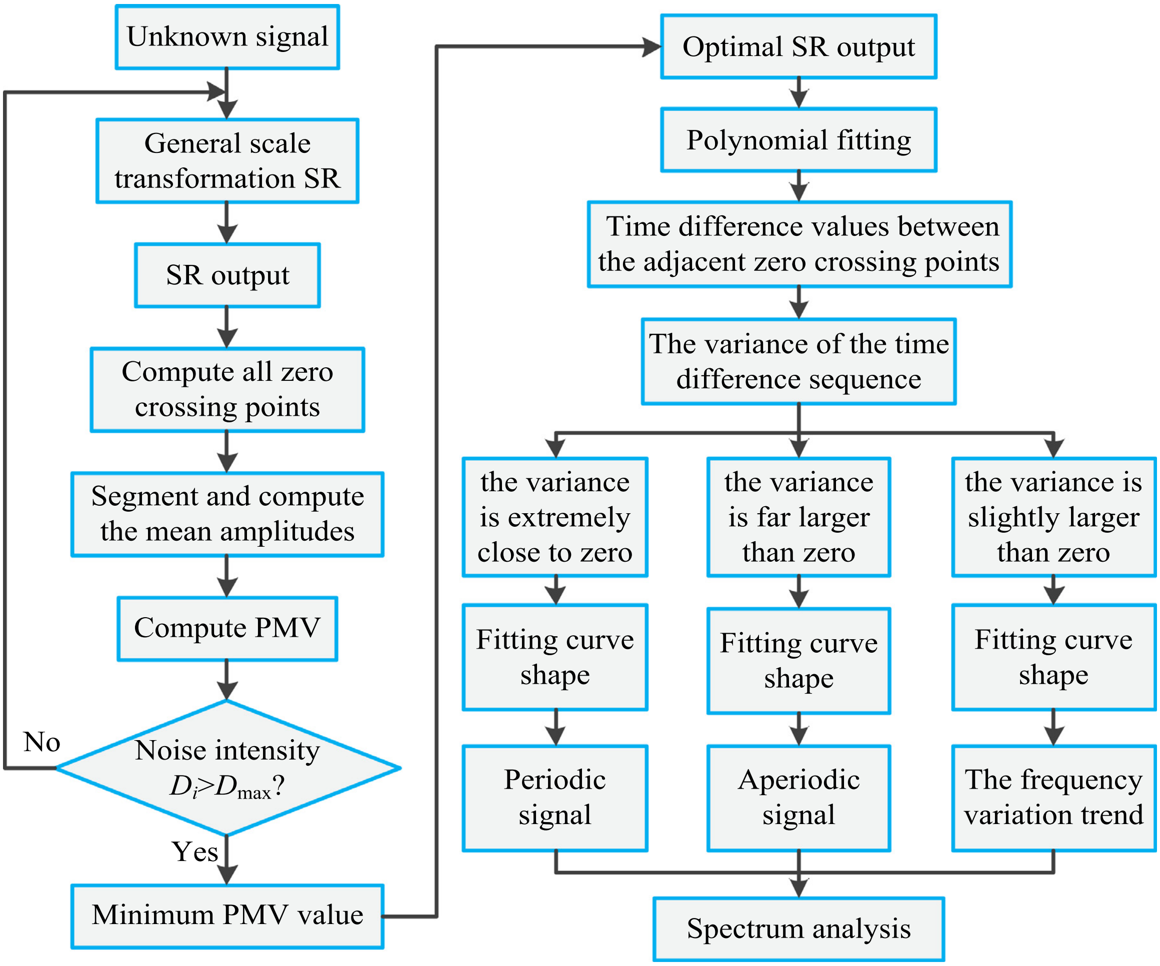

Fig. 4. The flowchart of realizing the PMV indicator and the parameters estimation strategy.

the SR method of general scale transformation [33] to directly process high frequency signals. The variable $S_j = \pm 1$ is a random variable with an independent distribution. Moreover, the width of the pulse is $Q = 20$, where $\beta = 1$ and $\Psi = 0$ are the frequency variation rate and initial phase. The discrete forms of the harmonic signal, binary signal and chirp signal shown in Eq. (3) are respectively realized at sampling frequency $f_{sh} = 100{,}000$ Hz, $f_{sb} = 100$ Hz and $f_{sc} = 1000$ Hz. These sampling frequencies satisfy the sampling theorem to ensure that the signals are not distorted during the sampling process.

The dependence of the PMV indicator on the noise intensity for different signals is shown in Fig. 5. For the extremely weak unknown signals, the nonlinear system is adopted to enhance the weak signals and convert the noise energy by employing a fourth-order Runge-Kutta algorithm, in which the calculation step is the reciprocal of the sampling frequency for each target signal. The existence of a minimum PMV value characterizes the optimal resonance state of the nonlinear system in each subfigure. The minimum PMV values are all close to zero for different signals, which show that all resonance responses are in form of the full-wave signals. It also displays that the energy accumulation of a Brownian particle in each potential well takes the same time, that is, the conversion rate at which a Brownian particle switches from one potential well to the other is equal. The recovery processes of the unknown signals are given under the most suitable noise intensities corresponding to the minimum PMV values in Figs. 6–8. According to the general scale transformation method [33], a scale coefficient m is required to realize the SR. For the harmonic signal and chirp signal, we respectively choose scale coefficients $m = 1500$ and $m = 100$ in accordance with Lorentz power spectrum of noise. Then, we choose a set of system parameter in scope [0, 2] for each signal. Further, we tune noise intensity to obtain the minimum PMV value, as shown in Fig. 5. The selected three sets of system parameters are (0.9, 1.8), (0.5, 1.5) and (0.6, 1.5), respectively. Hence, for the cases of the harmonic signal, binary signal and chirp signal, the nonlinear system parameters are $a = [1350, 0.5, 60]$ and $b = [2700, 1.5, 150]$, separately.

Three typical full-wave signals are contaminated by the strong Gaussian white noise in Figs. 6(b), 7(b) and 8(b) so that the features of the raw signals are abnormally difficult to be recognized, which create the states of the unknown signals. Aiming at these unknown signals, we intend to restore them as shown in Figs. 6(a), 7(a) and 8(a) from the strong noise

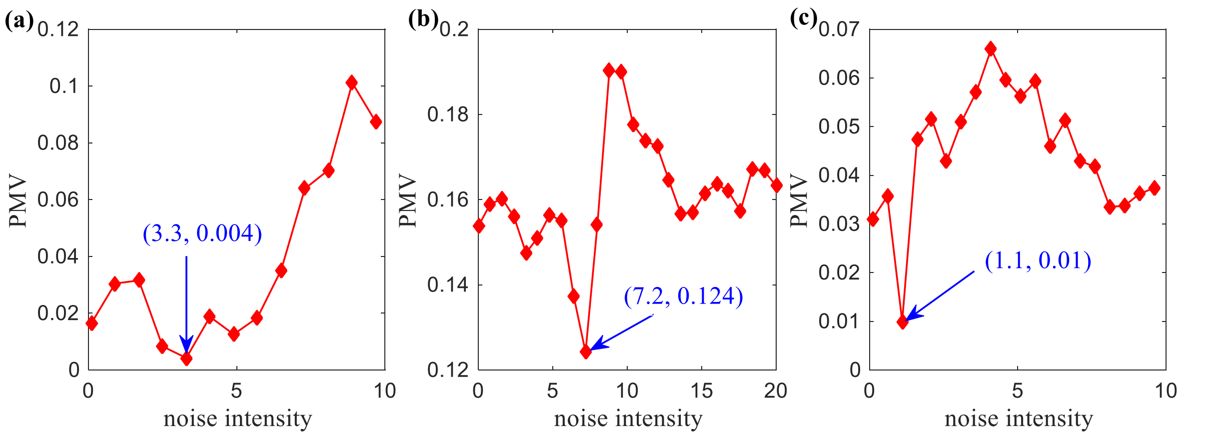

Fig. 5. The value of the PMV indicator depends on the noise intensity. (a) The signal to be restored is a harmonic signal. The minimum PMV is 0.004 when the noise intensity is $D = 3.3$. (b) The signal to be restored is a binary signal. The minimum PMV is 0.124 when the noise intensity is $D = 7.2$. (c) The signal to be restored is a chirp signal. The minimum PMV is 0.01 when the noise intensity is $D = 1.1$.

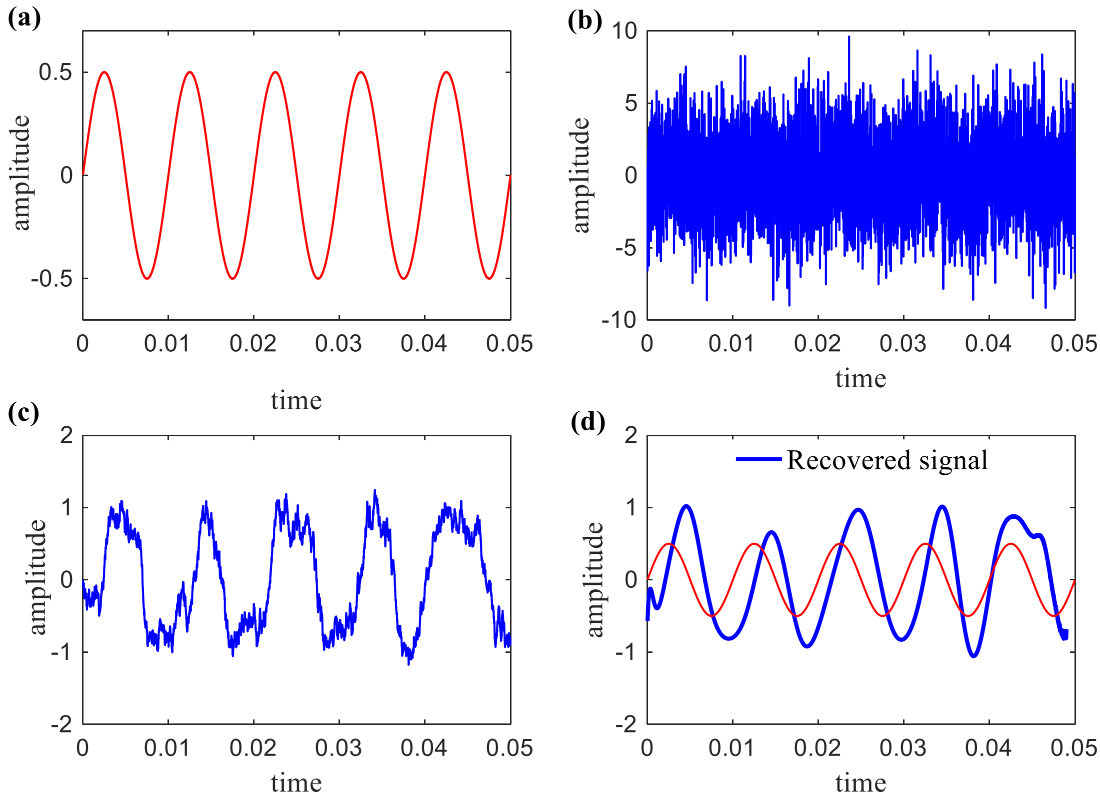

Fig. 6. The recovery of a harmonic signal. (a) The raw harmonic signal, (b) the time-domain spectrum of the signal submerged by a strong Gaussian white noise, (c) the time-domain spectrum of the resonance response of the nonlinear system, (d) recovered signal obtained by the polynomial fitting on the basis of subfigure (c).

background. Therefore, according to the foregoing introductions, the signals with strong noise are perceived as the excitations of the nonlinear bistable system to induce SR. The nonlinear system has an outstanding capability in exploiting noise energy and eliminating noise interference when the occurrence of nonlinear resonance. Figs. 6(c), 7(c) and 8(c) have given the output waveforms of the nonlinear system. It is not hard to see that all output waveforms are highly similar to the original input signals, which indicates that the minimum PMV criterion proposed in this paper can make the outputs of the nonlinear system to follow the characteristics of the input signals. Furthermore, the output waveforms are obviously amplified compared with the original signals. This proves that the nonlinear resonance is good at enhancing a weak signal

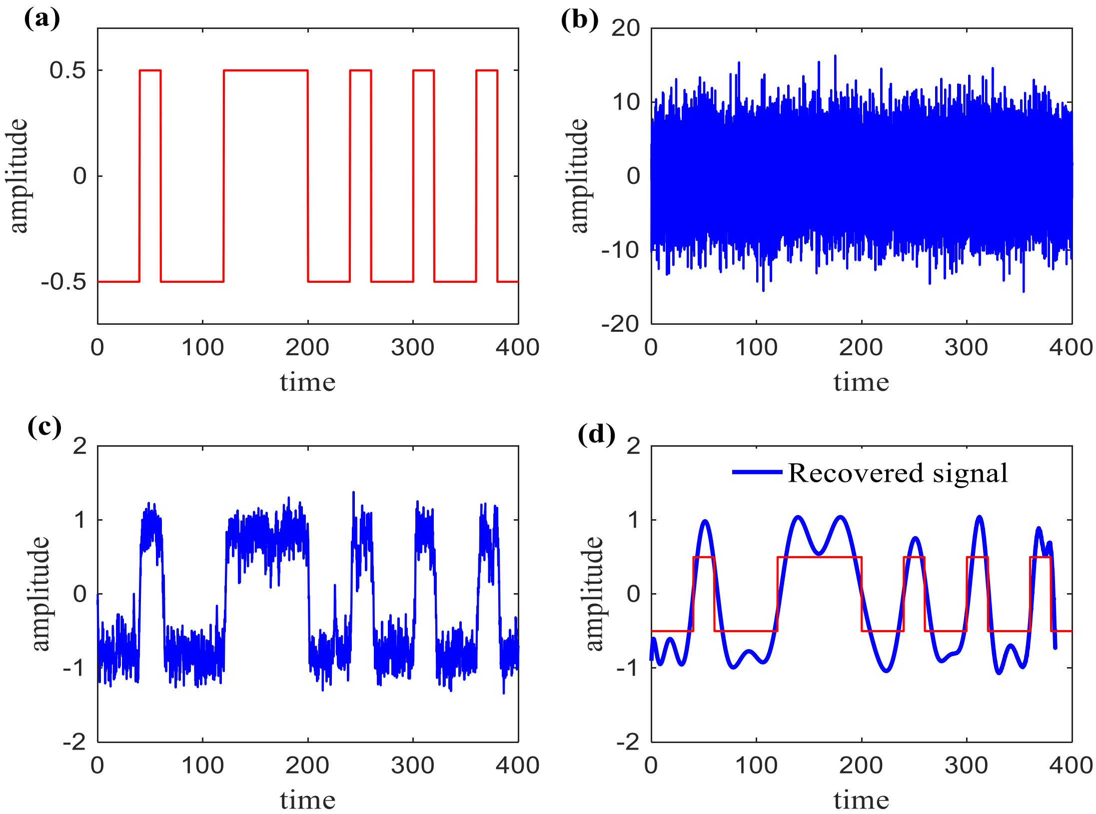

Fig. 7. The recovery of a binary signal. (a) The original binary signal, (b) binary signal totally buried by the strong Gaussian white noise, (c) the time-domain spectrum of the response of the nonlinear system, (d) recovered signal obtained by the polynomial fitting according to subfigure (c).

and weakening noise interference once again. In accordance with the output waveforms, the method of polynomial fitting is used to highlight the characteristics of the recovered signals.

The fitting forms of the recovered signals are displayed in Figs. 6(d), 7(d) and 8(d), which indicate that the recovered signals are all significantly strengthened compared to the original ones. From the time intervals between the adjacent zero crossing points of the fitting signals, the type of the recovered signals can be distinguished preliminarily. Time intervals reflect the related period information of the unknown signals. The approximate equal waveform intervals indicate that the unknown signal may be a harmonic signal in Fig. 6(d). The unknown signal may be an aperiodic square wave signal due to the irregular waveform intervals and some smaller cosine fluctuations in Fig. 7(d). The unknown signal may be an aperiodic signal with a changing frequency because the waveform interval decreases as the sampling time increases in Fig. 8(d).

To identify the features of unknown signals accurately, we calculate the time difference values between the adjacent zero crossing points of the fitting signals as shown in Figs. 9(a)–(c). In Fig. 9(a), as a result of the magnitude of the ordinate is abnormally small, which is 10^{-3}, and the variance of the time difference sequence is 1.1358×10^{-6} that is extremely close to zero, all the time difference values are approximately equal, what indicates that the recovered signal is a periodic signal. Furthermore, the mean of the time difference sequence is 0.005. The reciprocal of the twice mean is the same as the original signal frequency $f_H = 100\,\text{Hz}$. The unknown signal is absolutely recovered by means of the nonlinear resonance and the PMV indicator. Therefore, the unknown signal is a harmonic waveform. In Fig. 9(b), an obvious fluctuation of the time difference values represents the recovered signal without a clear periodic information. The variance of the time difference sequence is 556.78 that are far larger than zero. Hence, the unknown signal is a random aperiodic signal. Moreover, the time difference sequence between the recovered signal and the original signal coincides entirely showing that the recovered unknown signal is completely correct. In Fig. 9(c), the variance of the time difference sequence is 0.0046. With the increase of the sampling time, the time difference decreases gradually. The smaller the time difference is, the faster the frequency variation is. In addition, the time difference of the recovered signal is the same as that of the original signal indicating that the recovered unknown signal is accurate. To estimate the signal parameters, the reciprocals of the time difference values given in Fig. 9(c) are shown in Fig. 9(d), which denotes the variation tendency of the unknown signal frequency. The black point line is the fitting curve of the variable frequency for the unknown signal. The approximate gradient of the fitting curve is 0.5949, which equals $\beta/2$ approximately. These results indicate that the recovered signal is inerrant.

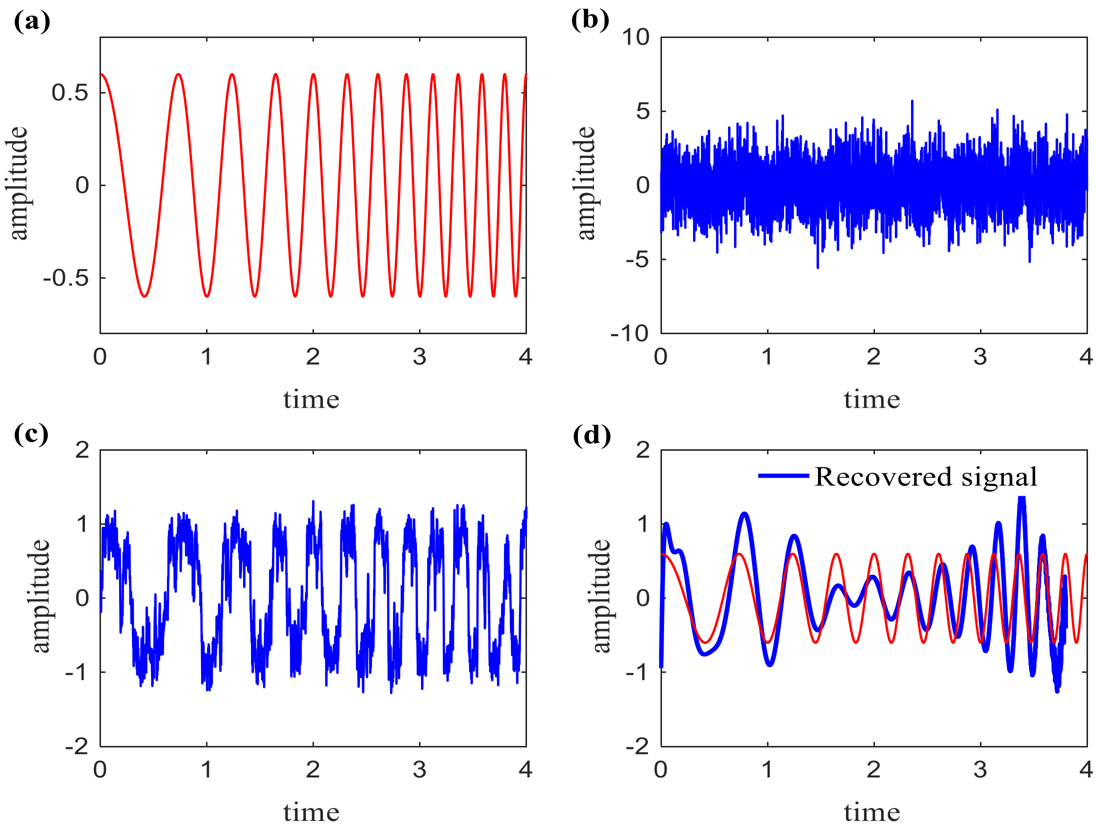

Fig. 8. The recovery of a chirp signal. (a) The raw chirp signal, (b) the chirp signal contaminated by a strong Gaussian white noise, (c) the output of the nonlinear system follows the original input signal under the minimum PMV criterion, (d) the recovered signal attained by the polynomial fitting on the basis of subfigure (c).

From the above analyses, we conclude that the unknown signals are individually periodic signal with the frequency $f_H = 100\,\text{Hz}$, random aperiodic signal, and linear frequency modulation signal with frequency change rate $\beta = 1.19$. They are the same as the original signals.

The above-mentioned analyses are performed in the time-domain response. The verification of the recovered signals is shown in Fig. 10 based on the frequency-domain analysis. The frequency spectrums of the recovered signals agree with the ones of the original signals, regardless of the positions and variation tendency of the feature frequencies in the frequency spectrums. Moreover, the amplitudes of the feature frequencies are all amplified indicating that the proposed approach and strategy has superior ability to restore unknown weak signals.

6. Conclusions

In conclusion, we have presented a method of recovering the unknown weak signal from a strong noise background. Different from all previous signal recovery methods, we regarded a strong noise coming from practice as a positive factor to assist the signal to be restored since a strong noise contains a considerable amount of energy. A nonlinear system excited by the unknown signal and a random noise can induce a SR phenomenon able to enhance a weak signal by exploiting abundant noise energy under the appropriate conditions. In particular, for a weak signal buried by a strong noise, the nonlinear resonance method is highly effective. Then, we have designed a new PMV indicator to quantify the quality of the SR response. We can find the optimal resonance of the nonlinear system by using a minimum PMV criterion. Designed PMV indicator is good at keeping the characteristics of an unknown signal in the process of SR. Based on the system output, a polynomial fitting and a parameter estimation strategy are applied to identify the unknown signal specifically. Three representative full-wave signals have been adopted to verify the validity and correctness of the proposed recovery method. All the results indicate that the unknown weak signals are enhanced and signal features are highlighted. Recovered signals are in full agreement with the original signals showing that the proposed method has excellent performance for the recognition of an unknown weak signal in a strong noise background.

Although this paper is implemented with a Gaussian white noise, the recovery method of the unknown signal is still applicable for the non-Gaussian white noise such as Poisson white noise, colored noise, bounded noise, etc. Hence, the proposed method is beneficial to the weak signal improvement and recovery in biological systems, circuit systems, and

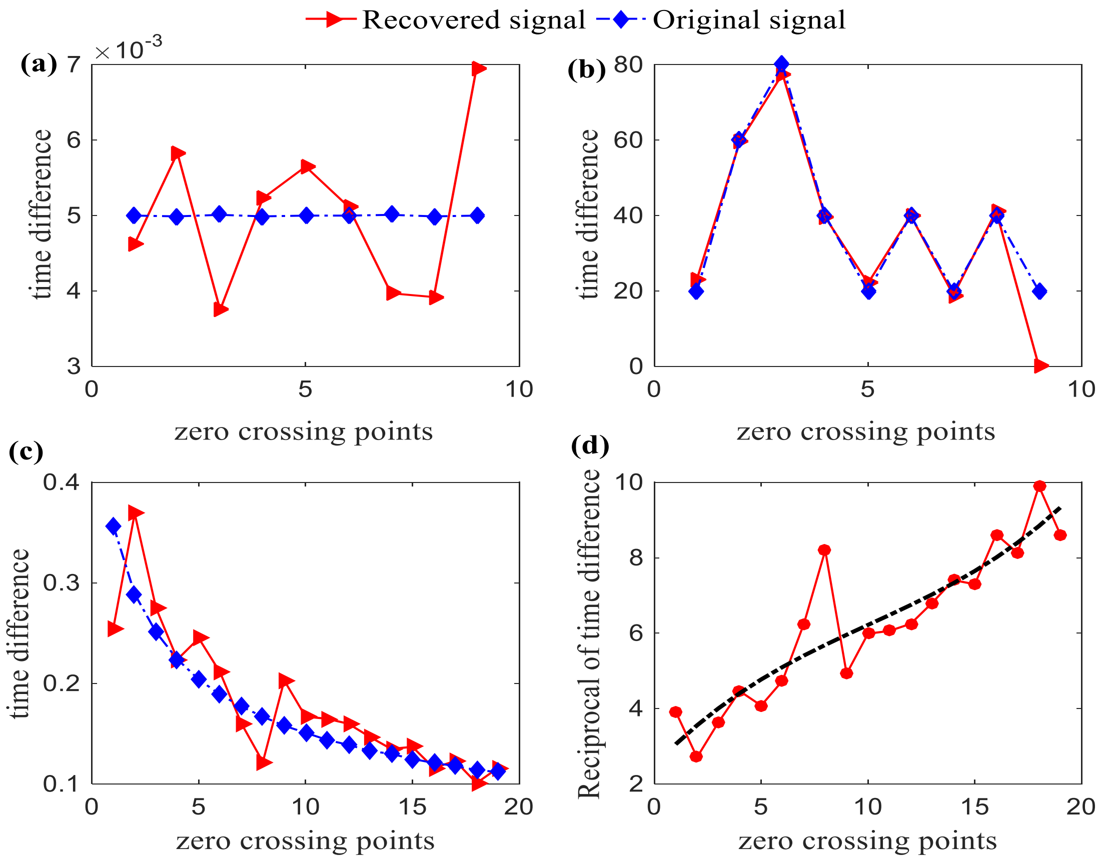

Fig. 9. The value of time difference depends on the zero crossing point. (a) Harmonic signal, (b) binary signal, (c) chirp signal, (d) The reciprocals of time difference values shown in subfigure (c).

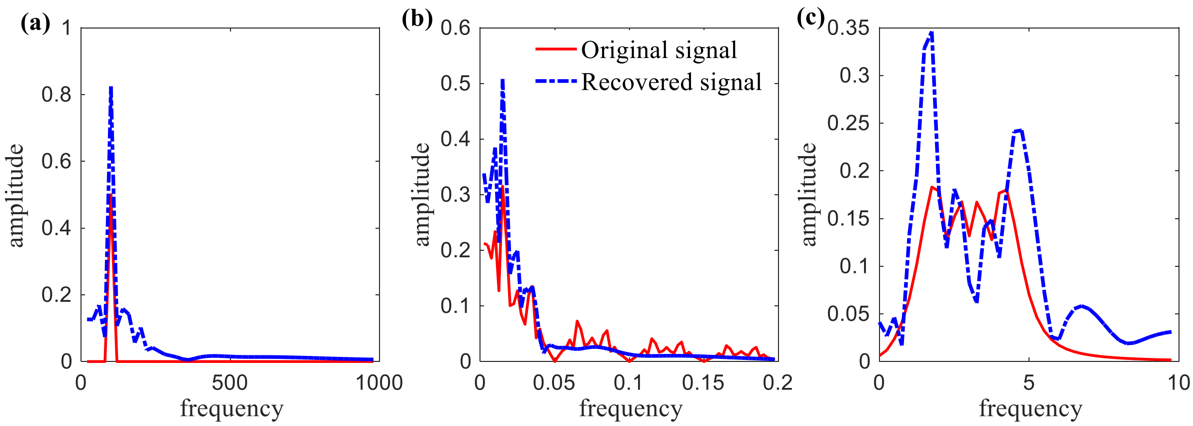

Fig. 10. The frequency spectrums of the recovered signals and original signals. (a) Harmonic signal, (b) binary signal, (c) chirp signal.

mechanical systems, among others. However, for the half-wave signal with unipolarity, the recovery method and the PMV indicator may not get the ideal recovery result in the bistable nonlinear system. The reason for this is that one of the potential wells of the bistable system is located in a positive region and the other in a negative region. Accordingly, particle oscillation can occur in only one potential well, which results in the weak half-wave signal cannot be distinctly enhanced and the strong noise cannot be adequately exploited. On the other hand, the resonance response of the nonlinear system depends on the system parameters. For uncontrollable noise in engineering practices, we can match the unknown noisy signal with optimal system parameters obtained by an intelligent optimization algorithm, which may attain the better effect in terms of the proposed method.

Acknowledgments

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